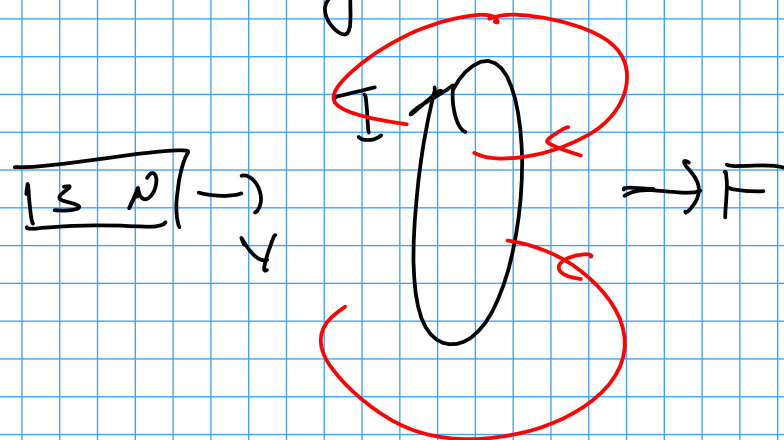
Is there work done generate B?

B does not do work $\quad \underbrace{\vec{F}}_{q\vec{v}\times\vec{B}} \cdot \vec{v} = \text{Power}$

To ans we need to know how to generate B

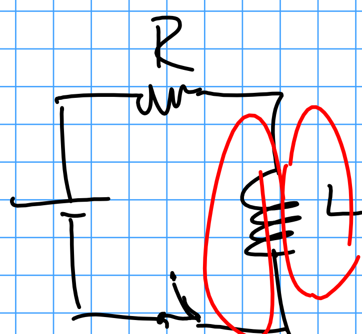
- Faradays

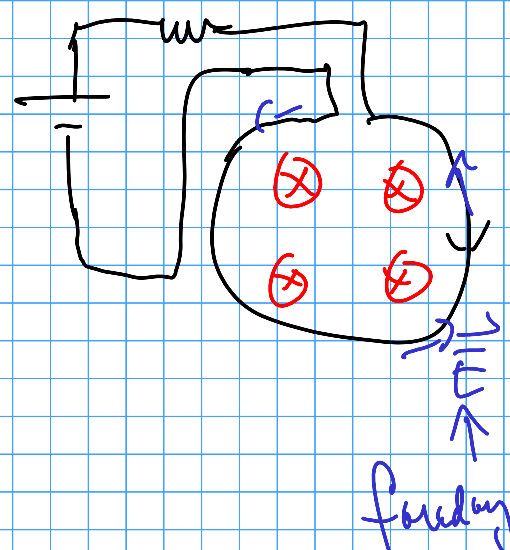


Complicated

Simple example that illustrates fundamental Principles

- Battery





$$W_{battery} = \xi\, \Delta V = \xi\, L \frac{dI}{dt}$$

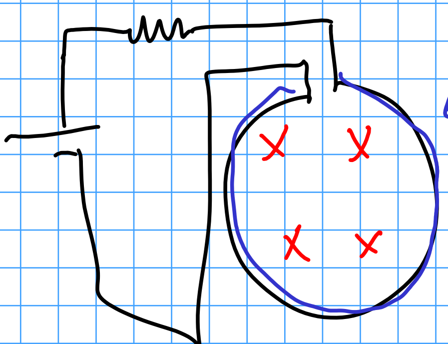
$$\Delta V = \mathcal{E}mf = -\frac{d\Phi_B}{dt}$$

$$\text{Power}_{batter} = I \Delta V = I L \frac{dI}{dt} = \frac{dW_{battery}}{dt}$$

$$W_{battery} = \frac{1}{2} L I^2 \longrightarrow \text{in terms of } \vec{B} \;\&\; \vec{A}$$

$$\Phi_{\text{one loop}} \propto I \qquad \Phi_{tot} = L I$$

$$\Phi = \int \vec{B} \cdot d\vec{a} = \int \vec{\nabla} \times \vec{A} \cdot d\vec{a} \xrightarrow{\text{stokes}} \oint \vec{A} \cdot d\vec{l}$$

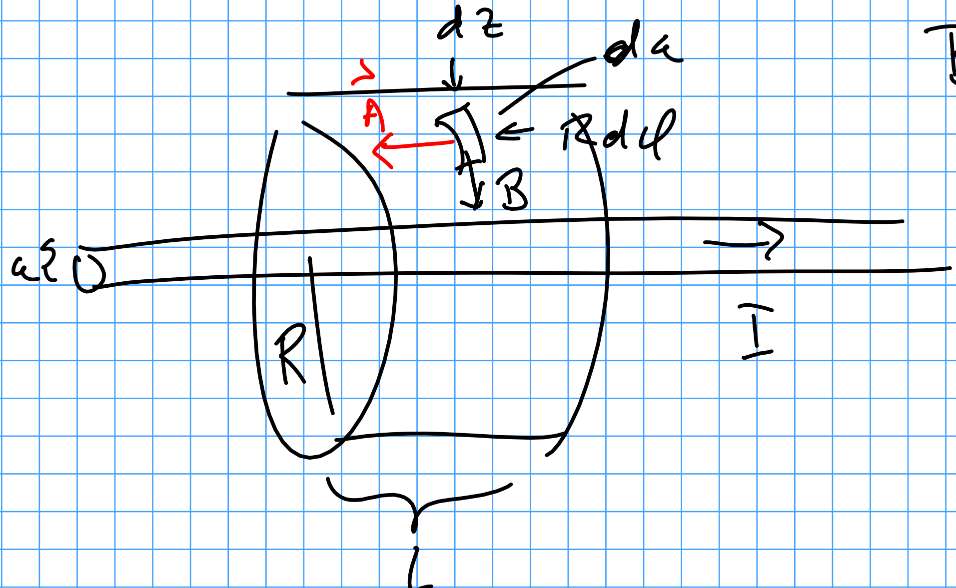


$$W_{batt} = \frac{1}{2} L I^2 = \frac{1}{2} I \underbrace{L I}_{\Phi} = \frac{1}{2} I \oint \vec{A} \cdot d\vec{l} = \frac{1}{2} \int \vec{A} \cdot \underbrace{I\, d\vec{l}}_{\vec{J}\, d\tau}$$

$$W_{batt} = \frac{1}{2}\int A\cdot\vec{J}\,d\tau \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\vec{J}_{disp}$$

$$\vec{J} \leftarrow \frac{\vec{\nabla}\times\vec{B}}{\mu_0}$$

$$W_{batt} = \frac{1}{2\mu_0}\left[\int_{vol} B^2\,d\tau - \oint \underbrace{\vec{A}\times\vec{B}\cdot d\vec{a}}_{\text{Int Surface}}\right]$$



$$B = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

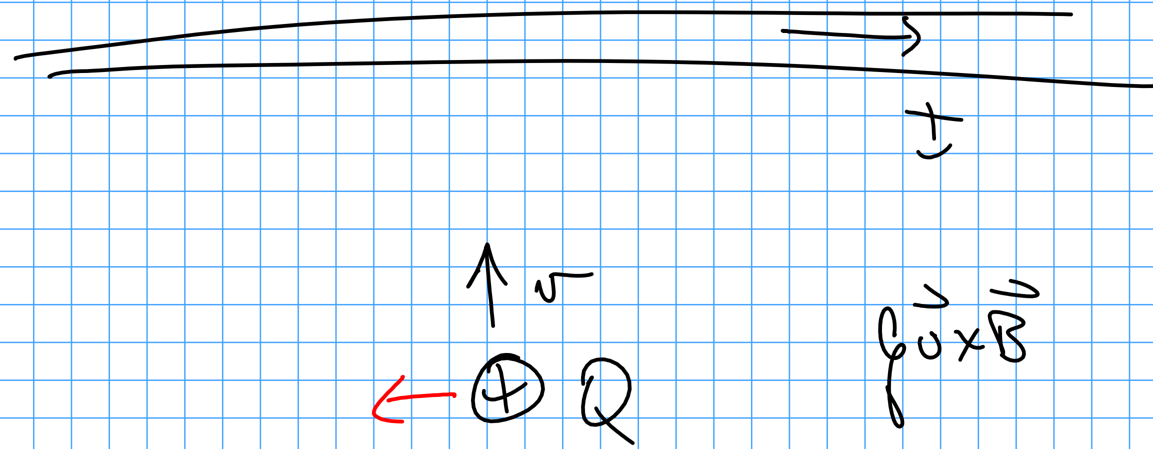
$$\vec{A} = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{r}{a}\right)\hat{z}$$

$$\vec{A}\times\vec{B}\cdot d\vec{a} = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{r}{a}\right)\frac{\mu_0 I}{2\pi r}\underbrace{\hat{z}\times\hat{\varphi}}_{(-\hat{r})}\,R\,d\varphi\,dz\cdot\hat{r}$$

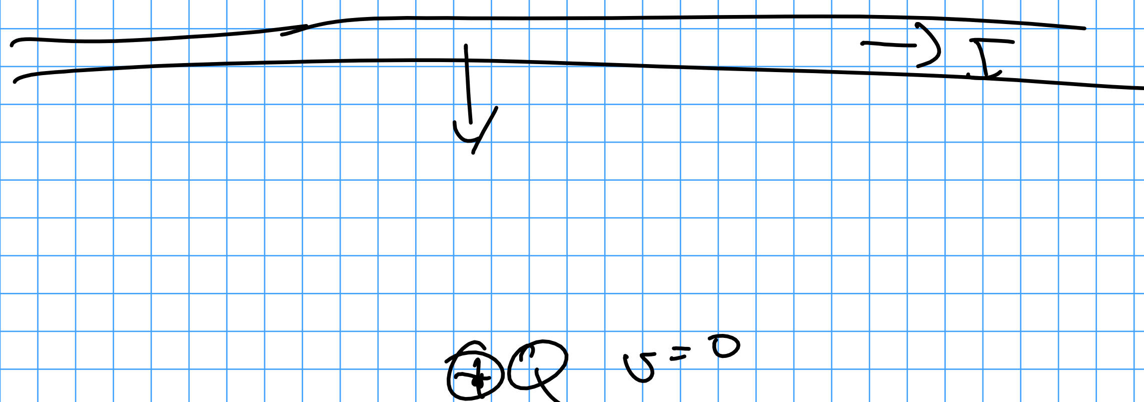
$$B^2\,d\tau = \left(\frac{\mu_0 I}{2\pi r}\right)^2\hat{\varphi}\cdot\hat{\varphi}\,R\,d\varphi\,dz\,dr$$

Same ans if $R\to\infty$ is $\displaystyle\int\frac{B^2}{2\mu_0}d\tau - \int\cancel{\vec{A}\times\vec{B}\cdot da}\to 0$

Ex:

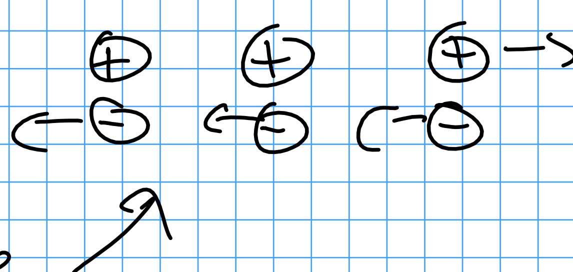


in frame of charge Q

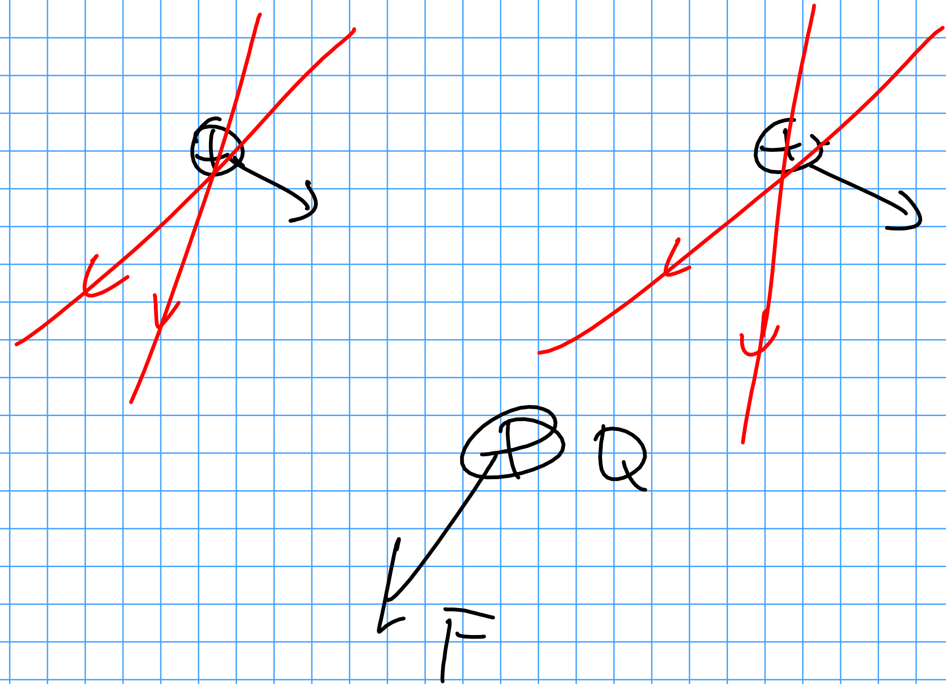


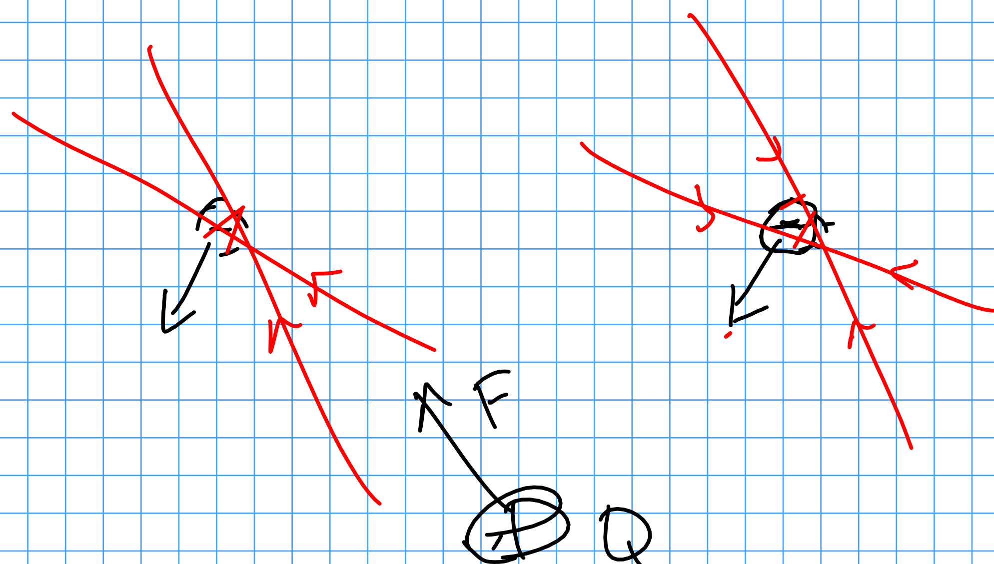
Problem Sol.

Simplify by letting I be



Go to frame of Q





Sum of forces from ⊕ & ⊖ charges