9/18

Note Title 9/18/2006

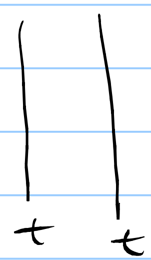
HW 2

$$T = \frac{T_L T_R}{1 - R_R R_L}$$

$T_L = T_R = t$

$R_L = R_R = r$

$$\boxed{T = \frac{t^2}{1 - r^2}}$$



T

$$T = t^2 + t^2 r^2 + t^2 r^4 + \cdots$$
$$= t^2 \left[1 + r^2 + r^4 \cdots \right]$$

physical vector mag. + direction.

independent of coordinate System

invariance

Abstract the properties and define new math. objects

Vector Space

[P.72 Scales' notes]

set of vectors, set of Scalars

Vectors $x, y \in V$

Scalars $\alpha, \beta \in F$

V1 $(x+y)+z = x+(y+z)$

V2 $x+y = y+x$

V5 $\alpha(x+y) = \alpha x + \alpha y$

V8 $1 \; x = x$

$I x = x$

$s = (s_1, s_2, s_3, s_4, \ldots s_{1000})$

$q = (q_1, q_2, \qquad q_{1000})$

Satisfy the properties of vector

$\vec{s}, \vec{q} \in R^{1000}$

$\vec{s} + \vec{q} \qquad \vec{s} \cdot \vec{q} = \|\vec{s}\| \, \|q\| \cos\theta$

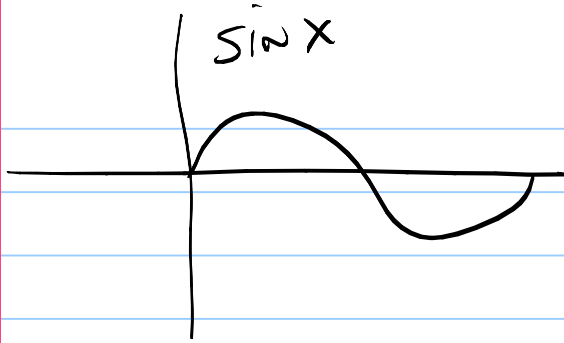
$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix}$

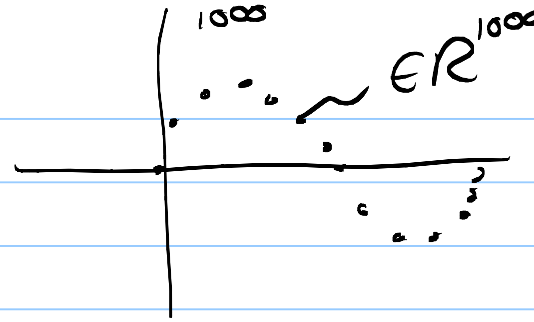
$A + B \qquad \alpha A$ matrices are vectors

$A, B \in R^{2\times 3}$

$f(x) \quad x \in [0,1]$ vectors

$g(x) \quad x \in [0,1]$





introduce notation

$\vec{x}, \vec{y} \in R^N$ dot. product, inner product

Einstein

$$\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = (\vec{x}, \vec{y}) = \sum_{i=1}^{N} x_i y_i = x_i y_i$$

summation conven

$A \in R^{N \times m}$ $x \in R^m$ $y \in R^N$

$$(A \cdot \vec{x})_i = \sum_{j=1}^{m} A_{ij} x_j$$

$$\vec{y}^T (A \cdot \vec{x}) = (\vec{y}, A\vec{x})$$

$$= \sum_i y_i \, [A\vec{x}]_i$$

$$= \sum_i y_i \left(\sum_j A_{ij} x_j \right)$$

$$= \sum_i \sum_j A_{ij} y_i x_j$$

$$= \sum_i \sum_j A^T_{ji} y_i x_j$$

$$= (A^T \vec{y}, \vec{x})$$

Q is an orthogonal matrix $\Leftrightarrow$

$$Q^T Q = QQ^T = I_N \qquad Q \in R^{N \times N}$$

$$[1] \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x \in R^N \qquad (Qx, Qx) = \|Qx\|^2$$

$$= (Q^T Q x, x) = (x, Q^T Q)$$

$$= (x, x) = \|x\|^2$$

$$\vec{x} \cdot \vec{x} = \sum_{i=1}^{N} x_i^2 = (\vec{x}, \vec{x}) = \|x\|^2$$

Suppose we want to measure

mass of an object

m = true mass

Goal to estimate m

$m_1, m_2, m_3 \ldots \quad \underset{\sim}{m} = \{m_i\}_{i=1}^{N}$

$$f_1(m) = \sum_{i=1}^{N} (m - m_i)^2$$

an estmate of $\underset{\sim}{m}$ is $\min_{m} f_1(m)$

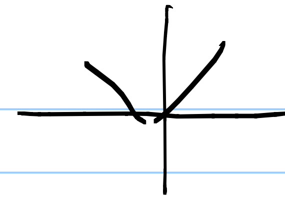
$$\frac{df_1}{dm} \Rightarrow 0 \qquad \cancel{2} \sum_{i=1} (m - m_i) = 0$$

$$\sum_{i=1}^{N} m = \sum_{i=1}^{N} m_i$$

$$= N \cdot m = \sum_{i=1}^{N} m_i$$

$$m_{est} = \frac{1}{N} \sum_{i=1}^{N} m_i$$

$$f_2(m) = \sum_{i=1}^{N} |m - m_i|$$



$$\frac{df_2}{dm} \Rightarrow 0 \quad = \sum_{i=1}^{N} \mathrm{sgn}(m - m_i) \quad = 0$$

$$\mathrm{sgn} = \frac{x}{|x|} = \pm 1$$

$$+1 + (-1) + (-1) + (+1) = 0$$

$$\Rightarrow m_{est} = \mathrm{median}(m_i)$$

goal $$\min_{m} \sum_{i=1}^{N} (m-m_i)^2$$

$$\Rightarrow \quad m = \text{mean}(m_i)$$

$$\min \sum_{i=1}^{N} |m-m_i|$$

$$\Rightarrow \quad m = \text{median}(m_i)$$