1) Two simple ways to measure laser beam size. Since a Gaussian beam does not have a well-defined edge, it is hard to know exactly the beam size by looking at it. One way is to calibrate a CCD camera, look at a lineout of the beam, and fit it to a Gaussian. There are two other ways to measure the spot size, both assume the beam has a Gaussian shape.
a. Knife-edge scan. Measure the power transmitted past a knife edge placed on a translation stage. You measure the positions for the knife edge that transmit $10 \%$ and $90 \%$ of the full power. Find an analytic relation between the difference between these two positions $x_{90-10}$ and the $1 / \mathrm{e}^{2}$ radius $w$ of the Gaussian beam. (This is especially useful for focused beams.)
b. Iris transmission: you center an iris (an aperture that can be changed in diameter) on a beam, and close the iris until one half of the total power is transmitted. Then you measure the diameter of the iris with calipers, $d_{1 / 2}$. Find the connection between this measured diameter and the $1 / \mathrm{e}^{2}$ radius $w$ of the beam. (This method is best for larger beams.)
2) Consider the energy level scheme shown in the figure below. Starting at $t=0$, where the atoms are in the ground state, the atoms are raised from level 0 to level 2 at a pump rate $R_{p}$. The lifetime of levels 1 and 2 are $\tau_{1}$ and $\tau_{2}$ respectively. Assuming that the ground state 0 is not depleted to any significant extent and neglecting stimulated emission:
a. Write the rate equations for the population densities, $N_{1}$ and $N_{2}$, of level 1 and 2 respectively;
b. Analytically calculate $N_{1}$ and $N_{2}$ as a function of time. If you want to check your work, you might use DSolve[ ] in Mathematica, but show the steps how to get the solution.
c. Use Mathematica to plot the population densities $\left(N_{1}(\mathrm{t})\right.$ and $\left.N_{2}(\mathrm{t})\right)$ and the population inversion $\left(N_{2}(\mathrm{t})-N_{1}(\mathrm{t})\right)$ on the same plot. Make two plots using the following two input values (you may pick an arbitrary value of $R_{\mathrm{p}}$ ):
i. $\tau_{1}=2 \mu \mathrm{~s}, \tau_{2}=1 \mu \mathrm{~s}$
ii. $\tau_{1}=1 \mu \mathrm{~s}, \tau_{2}=2 \mu \mathrm{~s}$

Hint: the differential equation for the population of level 1 can be solved by multiplying both sides by the factor $\exp \left(\mathrm{t} / \tau_{1}\right)$. In this way the left-hand side of the preceding differential equation becomes a perfect differential.

3) Suppose the peak cross-section for absorption on a particular transition is $\sigma_{0}=10^{-16} \mathrm{~cm}^{2}$ at 500 nm . The transition has a Lorentzian lineshape with a FWHM of $\Delta v_{t}=50 \mathrm{GHz}$.
a. Write an expression for the spectrally dependent cross-section $\sigma(v)$ in terms of $\sigma_{0}$ and $\Delta \nu_{t}$. Make sure your expression has the correct dimensions.
We illuminate with a light source and we want to calculate the absorption rate under different conditions.
b. In this part, assume that the light source also has spectral intensity with a Lorentzian lineshape with a FWHM of $\Delta \nu_{s}$. Calculate an analytic expression for the total transition rate $W_{12}=\frac{1}{h v_{0}} \int \sigma(v) I(v) d v$. Hint: You may use Mathematica to help on the integral if you wish. Use Integrate[ ], with the Assumptions option to let it know that the variables are real (e.g. $\operatorname{Im}[x]==0$ ) and, if this is the case, positive $(x>0)$.
c. The calculation takes an especially simple form if one of the lineshapes is much narrower than the other. Simplify the expression for this limit.
d. Explain how this approximation can be applied to estimate the integral when the lineshape functions are arbitrary and have different center frequencies, but you know that one is much smaller than the other. A sketch of the two functions should be helpful.

