Homework 4:

1) a) Show using Maxwell's Eqns and the definitions from Chilwell's paper of u, v, w that

$$v = \frac{\gamma}{ik\alpha}\frac{du}{dx} \quad ; \quad w = \frac{\beta\gamma}{\alpha}u \quad ; \quad u = \frac{1}{ik\gamma\alpha}\frac{dv}{dx}$$

for <u>both</u> TE and TM polarization.

b) Using the boundary conditions we already know for $\vec{E}$ and $\vec{B}$, solve for boundary conditions at an interface for u, v, w. Once again, do this for <u>both</u> polarizations.

c) For a right going wave (positive k_x), show

$v^+ = \gamma u^+$ (or) $\begin{pmatrix} u^+ \\ v^+ \end{pmatrix} = \begin{pmatrix} 1 \\ \gamma \end{pmatrix} u^+$

show for the left going wave

$\begin{pmatrix} u^- \\ v^- \end{pmatrix} = \begin{pmatrix} 1 \\ -\gamma \end{pmatrix} u^-$

d) Use the fact that

(1) u can be written a $u = u^+ + u^-$ (or) $\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ \gamma \end{pmatrix} u^+ + \begin{pmatrix} 1 \\ -\gamma \end{pmatrix} u^-$

$$\left.\begin{aligned} &(2)\; u^+(x_{j-1}) = u^+(x_j)\, e^{-k\alpha_j(x_j - x_{j-1})} \\ &\quad\; u^-(x_{j-1}) = u^-(x_j)\, e^{+k\alpha_j(x_j - x_{j-1})} \end{aligned}\right\}$$ <u>Explain</u> why these are true.

[Define $\Phi_j = k\alpha_j(x_j - x_{j-1})$]

(3) $e^{\pm \underbrace{ik\alpha_j(x_j - x_{j-1})}_{\Phi_j}} = \cos(\Phi_j) \pm i\sin(\Phi_j)$

to show that $\begin{pmatrix} u_{j-1} \\ v_{j-1} \end{pmatrix} = M_j \begin{pmatrix} u_j \\ v_j \end{pmatrix}$ where

$$M_j = \begin{pmatrix} \cos\Phi_j & -\frac{i}{\gamma_j}\sin\Phi_j \\ -i\gamma_j \sin\Phi_j & \cos\Phi_j \end{pmatrix}$$

2) a) Write down r_{cs} and t_{cs} from the paper.

b) Write down R and T from the paper and explain what they are.

c) Write down the time averaged power flows from the paper.

d) Write down Φ_{cs} from the paper, and explain its physical significance.

e) Show that if there is only a simple interface (no layering, which means $M = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$) that T and R reduce to what we solve for in class. Show it works for both TE and TM waves.

3) Using the Mathematica code, create a 1/4 wave stack consisting of a glass ($n=1.5$) cover and substrate, and SiN layers ($n=1.8$) inside. Make the 1/4 wave stack optimal for normal incidence and yellow light ($\lambda_0 = 580\,nm$). Remember $\lambda_{glass} = \frac{\lambda_0}{n}$, etc.

a) Calculate R, T for $m = 2, 10, 100$, where m is the number of repeats of SiN / Glass.

b) Find m where R is first above 90%.

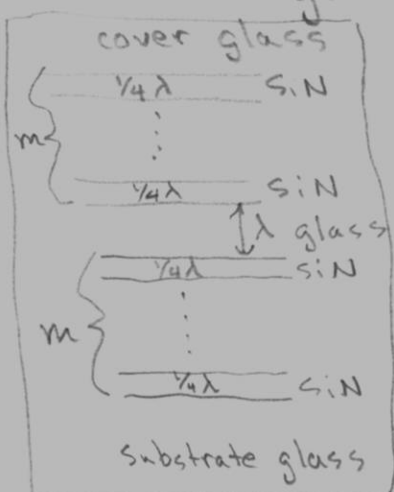
c) For the m in part (b), create a graph of R, T (on the same axis) as you vary the wavelength (without changing thicknesses) from 300 nm to 800 nm (the whole visible spectrum). Is this a good reflector for the whole range (could it be a mirror?)?

d) For the m in part (b), create a graph of R, T varying the incident angle ($\lambda_0 = 580\,nm$ again) from $0°-90°$. Does this mirror work well off-axis?

4) a) Using the 1/4 wave stack from (3), find m for 99% reflection.

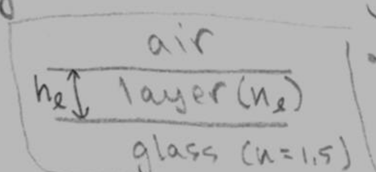
b) Now create two of those stacks on top of each other. Basically make $m = 2m$. What is R now?

c) Do the same as in (b), but this time make a space between the first m stacks and the second m stacks that is not 1/4 wavelength, but 1 wavelength.



What is R and T now? What you have just made is a resonant cavity. We'll learn more about that later.

5) a) Say you have a glass substrate and you want to eliminate reflection at normal incidence with a single layer on the glass. So your structure looks like



. Analytically solve for M in terms of constants, n_e and h_e. Then set $r_{cs} = \emptyset$ to find a relationship for n_e and h_e. Use normal incidence. This is an anti-reflection coating.

b) Make a graph for one case with set λ_0 varying θ_c ($0 \rightarrow 90°$).