Mode locking and ultrafast optics.

- in transient response (relaxation osc., Qswitching) we assumed dynamics were longer than τ_{RT}

- now look at fast time scales.

Outline:

t-w domains and Fourier transforms
- transform pairs

dispersive propagation, group velocity.

comb (or array) theorem

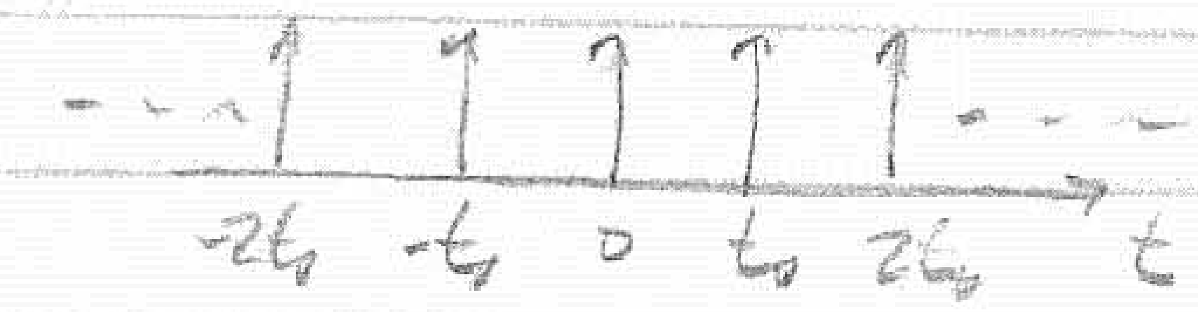
mode-locked pulse train.

phasing modes with a modulator.

Array theorem

comb function:

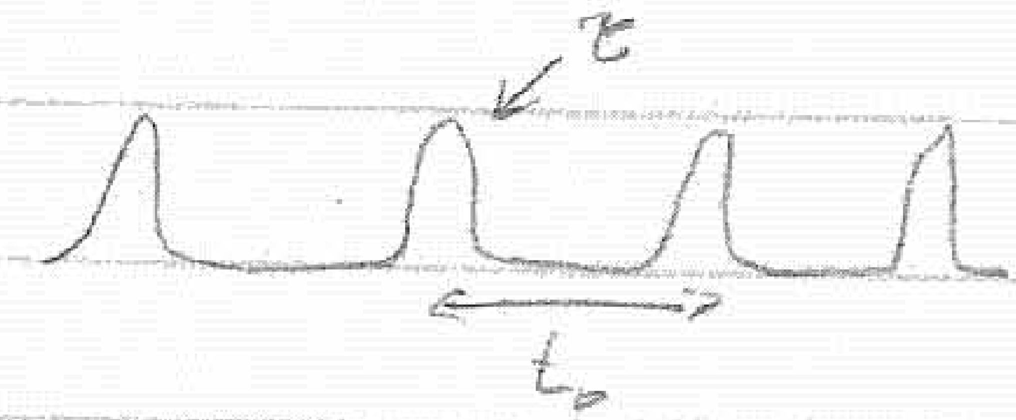
$$\text{comb}(t/t_0) = \sum_{n=-\infty}^{\infty} \delta(t - n t_0)$$



applications:

pulse train:

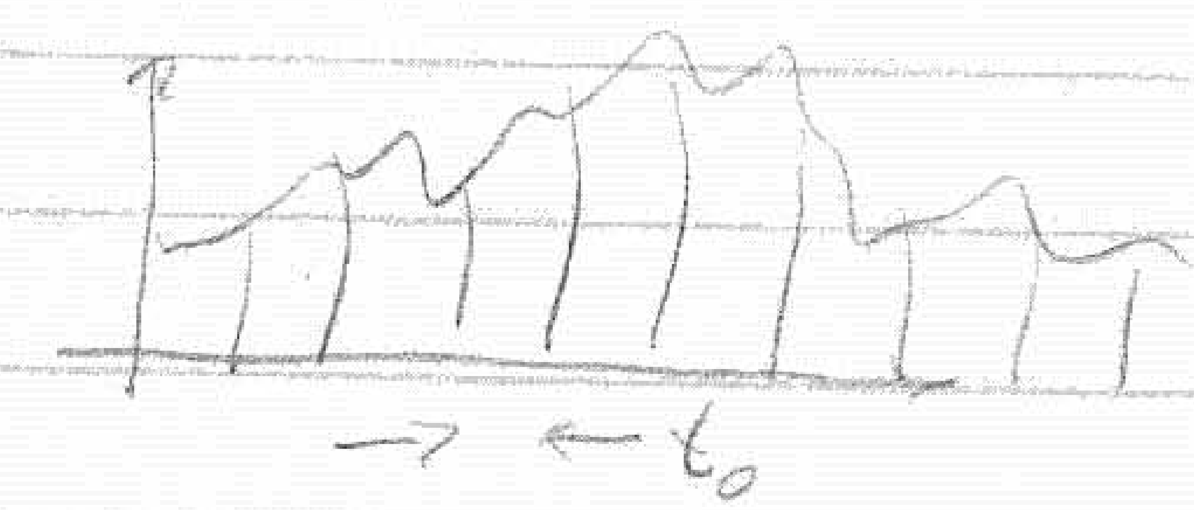
$$e^{-t^2/\tau^2} \otimes \text{comb}(t/t_0) \rightarrow$$



sampling:

$$g(t) \cdot \text{comb}(t/t_0)$$

t_0 = sampling period



grating

$$\left[a(x) \otimes \text{comb}(x/x_0)\right] \cdot \text{rect}(x/D)$$

groove shape — periodicity — D = grating length.

transform:

$$f(t) = \sum_{n=-\infty}^{\infty} \delta(t - n t_0)$$

$$F(\omega) = \sum_{n=-\infty}^{\infty} \mathcal{F}\{\delta(t - n t_0)\} = \sum e^{-i\omega t_0 \cdot n}$$

this is actually a comb function:

comb( ) is periodic, so write as Fourier series:

$$f(t) = \sum_n c_n e^{i2\pi n t/t_0}$$

coeff: $$c_n = \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} f(t)\, e^{-i2\pi n t/t_0}\, dt = \frac{1}{t_0}\int_{-t_0/2}^{t_0/2} \delta(t)\, e^{-i2\pi n t/t_0}\, dt$$

let limits $\to \pm\infty$

$c_n = 1/t_0$

$\therefore$ we can write

$$\text{comb}(t/t_0) = \frac{1}{t_0}\sum_n e^{i2\pi n t/t_0}$$

$$\mathcal{F}\{\text{comb}(t/t_0)\} = \frac{1}{t_0}\sum_n \mathcal{F}\{e^{i\frac{2\pi n}{t_0}t}\}$$

$$= \sum_n \frac{2\pi}{t_0}\delta(\omega + \frac{2\pi n}{t_0})$$

$$= \frac{2\pi}{t_0}\,\text{comb}\left(\omega/(2\pi/t_0)\right)$$

Mode-locked pulse train

Start in frequency space:

assume a Gaussian gain bandwidth $e^{-(\omega-\omega_0)^2/\Delta\omega_L^2}$

cavity $\rightarrow$ $\text{comb}(\omega/\Delta\omega_c)$ (ideal)

spectrum: $F(\omega) = \left(e^{-(\omega-\omega_0)^2/\Delta\omega_L^2}\right) \cdot \text{comb}\left(\omega/\Delta\omega_c\right)$

$= G(\omega) \cdot H(\omega)$

in time domain:

$$f(t) = g(t) \otimes h(t)$$

$$g(t) = \mathcal{F}^{-1}\left\{e^{-(\omega-\omega_0)^2/\Delta\omega_L^2}\right\} = e^{-i\omega_0 t}\,\mathcal{F}\left\{e^{-\omega^2/\Delta\omega_L^2}\right\}$$

$$= \frac{1}{\sqrt{\pi t_p^2}}\, e^{-i\omega_0 t}\, e^{-t^2/t_p^2} \qquad t_p = 2/\Delta\omega_L$$

$$h(t) = \mathcal{F}^{-1}\left\{\text{comb}\left(\omega/\Delta\omega_c\right)\right\}$$

since $\mathcal{F}\left\{\text{comb}(t/t_0)\right\} = \left(\frac{2\pi}{t_0}\right) \text{comb}\left(\omega/(2\pi/t_0)\right)$

let $\Delta\omega_c = 2\pi/t_0$ $\quad \Delta\nu_c = 1/t_0 \quad t_0$ = roundtrip time.

$\rightarrow h(t) = \frac{1}{\Delta\omega_c}\,\text{comb}(t/t_0)$

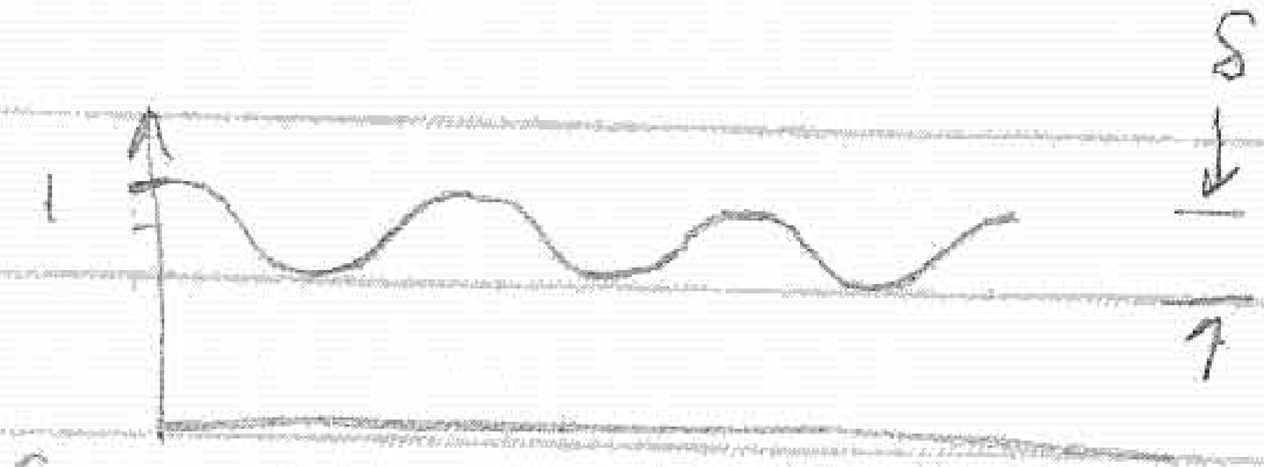
$$f(t) = g(t) \otimes h(t) = \frac{1}{\sqrt{4\pi}}\,\frac{\Delta\omega_L}{\Delta\omega_c}\left(e^{-i\omega_0 t}\, e^{-t^2/t_p^2} \otimes \text{comb}(t/t_0)\right)$$

Amplitude modulation modelocking:

Devices (active)

E-O modulation - Kerr cell, rotates polarization.

$$E_{out}(t) = E_{in}(t)\left[1 - \frac{\delta}{2}(1-\cos\omega_m t)\right]$$

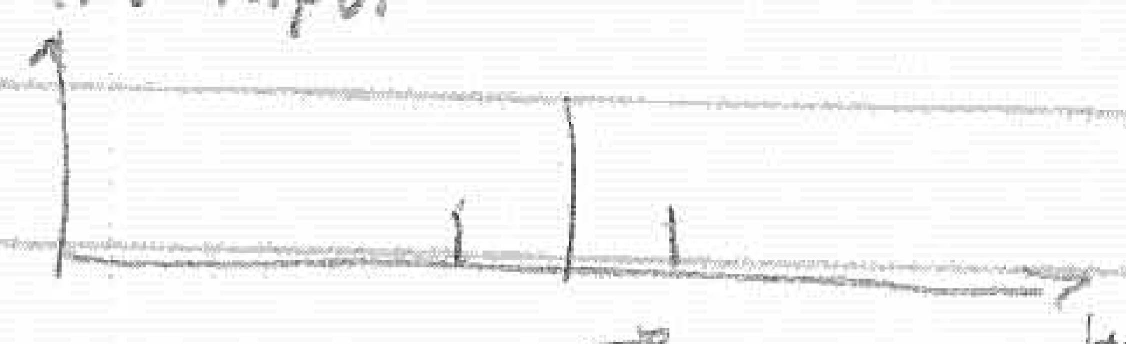


$$E_{out}(\omega) = E_{in}(\omega)\left(1-\frac{\delta}{2}\right) + \frac{\delta}{2}\,\mathcal{F}\{E_{in}(t)\cdot\cos\omega_m t\}$$

$$= \quad '' \quad + \frac{\delta}{2}\cdot\frac{1}{2\pi}E_{in}(\omega)\otimes\frac{1}{2}\left(\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right)$$

monochromatic input

→



ω_m = modulation frequency

sidelobes are phase-coherent with main wave.

IF $\omega_m = 2\pi\cdot\Delta\nu_c$ each longitudinal mode seeds its neighbor with an in phase signal.

→ longitudinal modes are locked in phase.

Time domain picture:

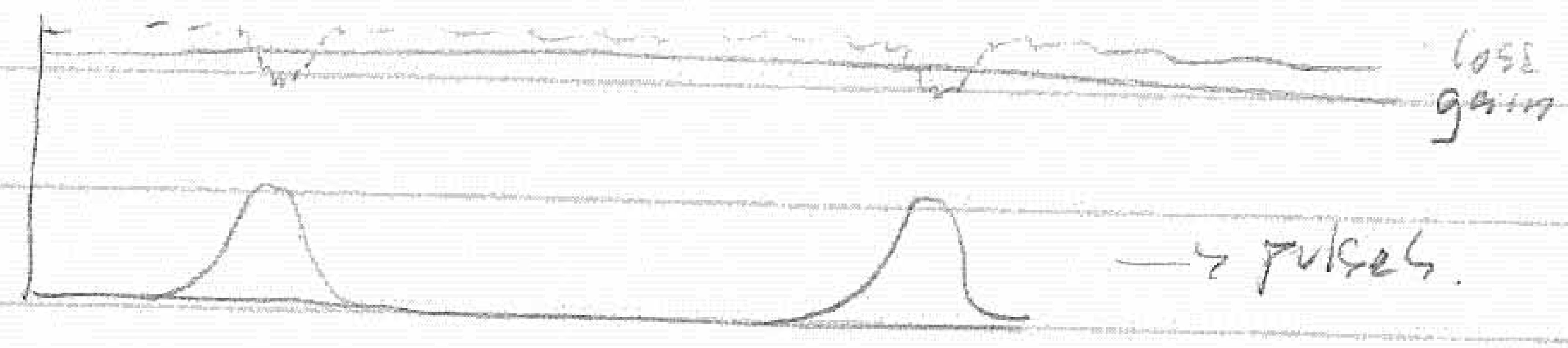
AM → time varying loss

when light is synched with low loss → higher gain.

Passive modelocking.

1) fast saturable absorber. (response, recovery time)

$$\alpha(I) = \alpha_0 \frac{1}{1 + I/I_s} \quad \text{for } I/I_s \ll 1 \quad \alpha \approx \alpha_0 (1 - I/I_s)$$



solution → $E(t) \sim \text{sech}(t/t_p)$