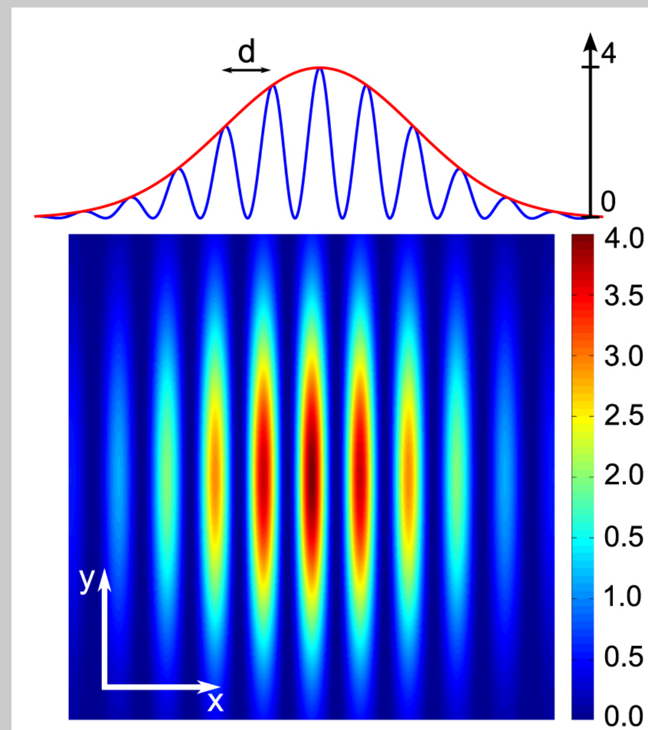
Laser beam propagation

- Main distinguishing characteristic of laser beams: **coherence!**
  - Allows the beam to be directional
  - Interference is easily observed
  - Phase is crucial

- Any transverse confinement of the beam leads to diffraction
  - Beams can be described either as a superposition of plane waves or of transverse modes



Laser beam propagation: outline

- Interference
  - Plane waves
  - Diverging waves
  - Interferometers: Michelson, Fabry-Perot
- Ray tracing with ABCD matrices
- Diffraction and Gaussian beams
  - Beam propagation with ABCD matrices
  - High-order modes
- Resonator design

Interference: ray and wave pictures

- Interference results from the sum of two waves with different *phase*:

$$E_{tot}\left(\Delta\phi\right)=E_1e^{ikz}+E_2e^{ikz+\Delta\phi}$$

- We measure intensity, which leads to interference

$$I_{tot}\left(\Delta\phi\right)\propto\left|E_1e^{ikz}+E_2e^{ikz+\Delta\phi}\right|^2=\left|E_1+E_2e^{i\Delta\phi}\right|^2$$

$$=I_1+I_2+\sqrt{I_1I_2}e^{i\Delta\phi}+\sqrt{I_1I_2}e^{-i\Delta\phi}$$

$$=I_1+I_2+2\sqrt{I_1I_2}\cos\left(\Delta\phi\right)$$

  - For equal intensity $I_1 = I_2$,

$$I_{tot}\left(\Delta\phi\right)=2I\left(1+\cos\left(\Delta\phi\right)\right)=4I\cos^2\left(\Delta\phi/2\right)$$

- How to generate, calculate phase difference?

Crossed beam interference

- Phase difference comes from relative tilt $\Delta\phi = 2kx\sin\theta$

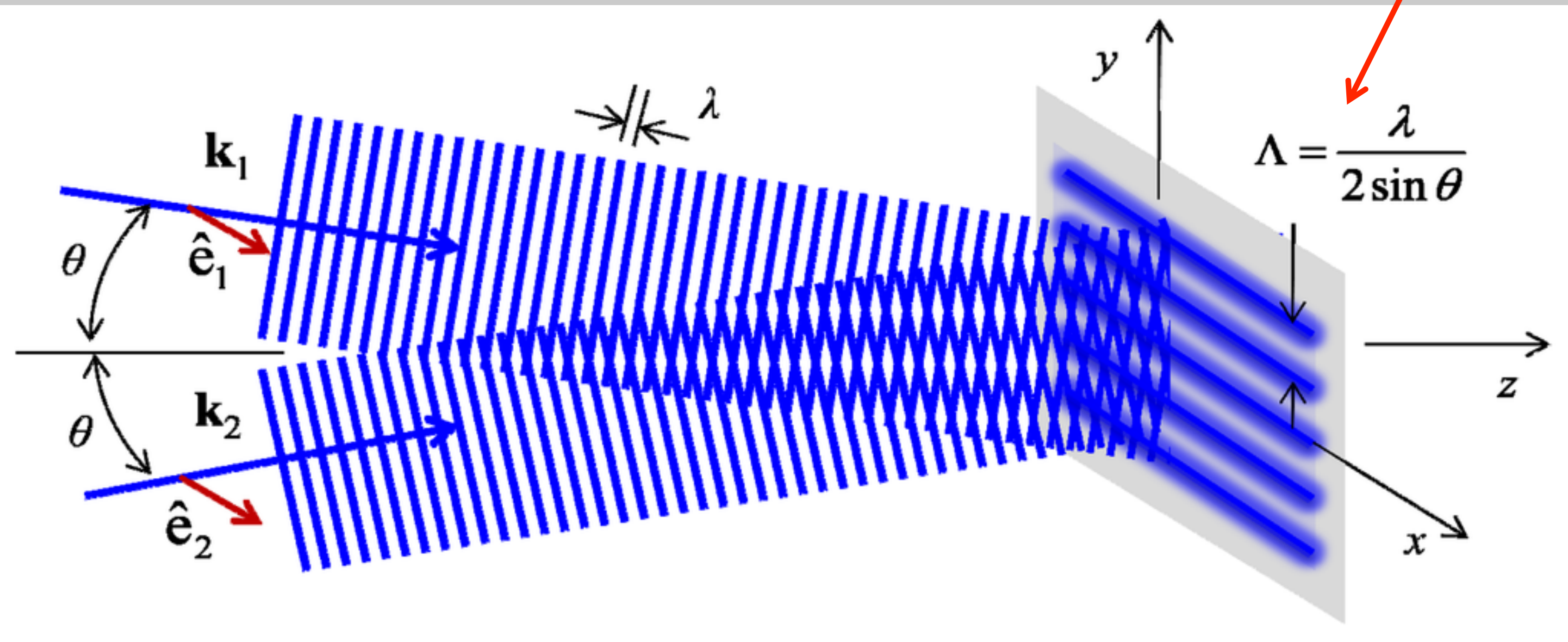
$$I_{tot}(\theta) \propto \left| E_1 e^{ik(x\sin\theta + z\cos\theta)} + E_2 e^{ik(-x\sin\theta + z\cos\theta)} \right|^2 = \left| E_1 e^{ikx\sin\theta} + E_2 e^{-ikx\sin\theta} \right|^2$$

$$= I_1 + I_2 + \sqrt{I_1 I_2}\, e^{2ikx\sin\theta} + \sqrt{I_1 I_2}\, e^{-2ikx\sin\theta}$$

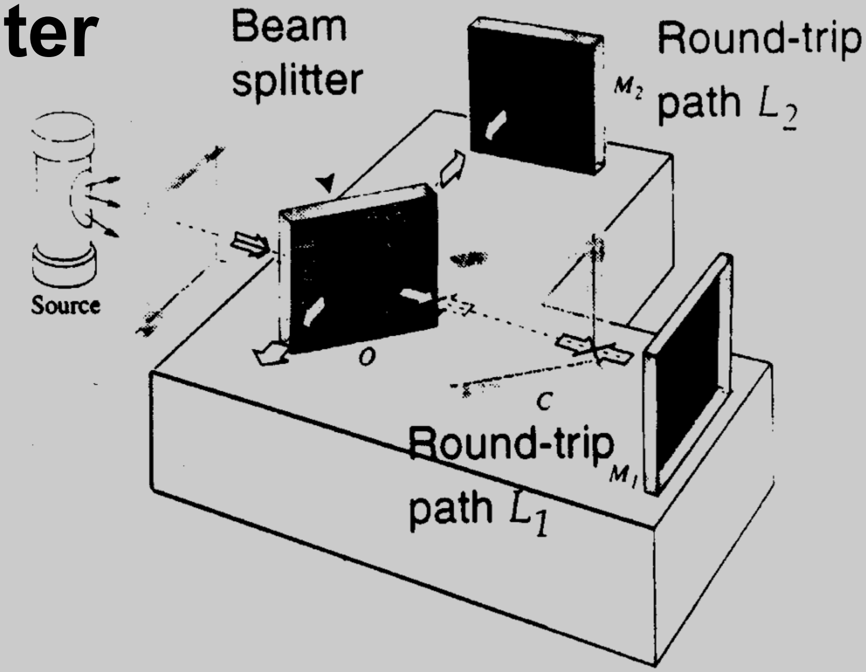
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(2kx\sin\theta)$$

Calculate fringe spacing

$\Delta\phi = 2\pi = 2k\Lambda\sin\theta$



The Michelson Interferometer



The Michelson Interferometer splits a beam into two and then recombines them at the same beam splitter.

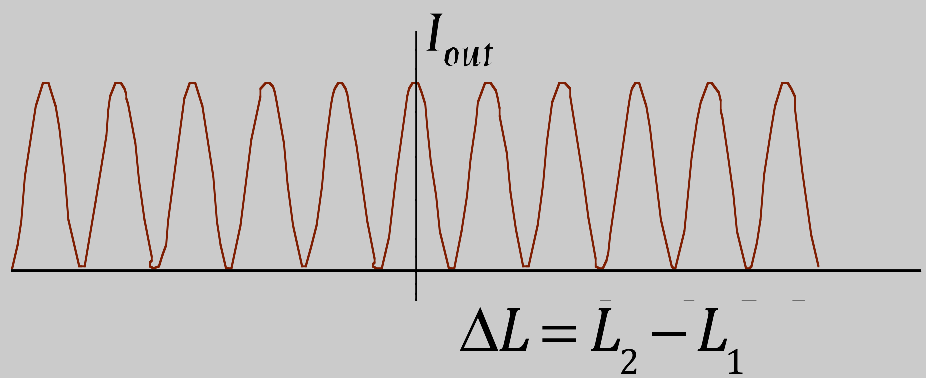
For plane wave input beam:

$$I_{out} = I_1 + I_2 + c\varepsilon\,\mathrm{Re}\left\{ E_0 \exp\left[i(\omega t - kz - kL_1) \right] E_0^* \exp\left[-i(\omega t - kz - kL_2) \right] \right\}$$

$$= I + I + 2I\,\mathrm{Re}\left\{ \exp\left[ik(L_2 - L_1) \right] \right\} \qquad \text{since } I \equiv I_1 = I_2 = (c\varepsilon_0 / 2)\left| E_0 \right|^2$$

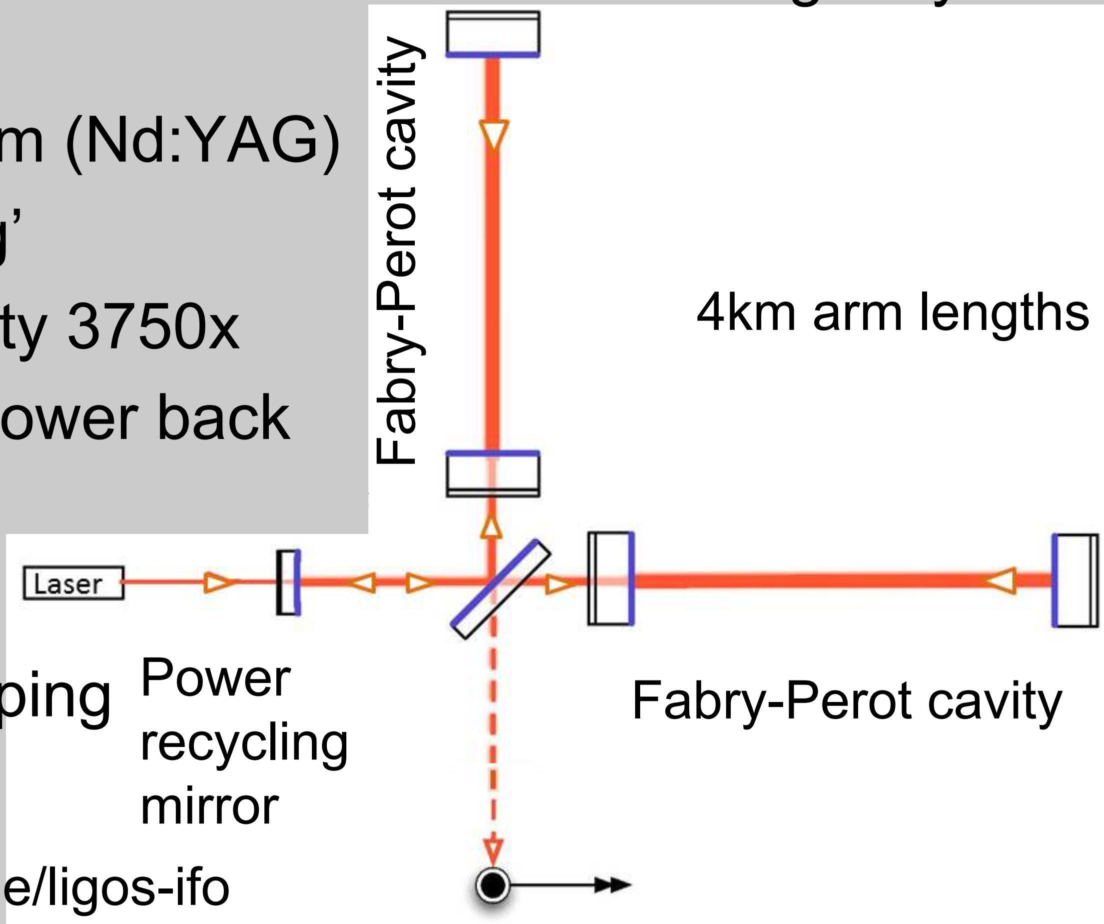
$$= 2I\left\{ 1 + \cos(k\Delta L) \right\}$$

Fringes (in delay)



The LIGO interferometer

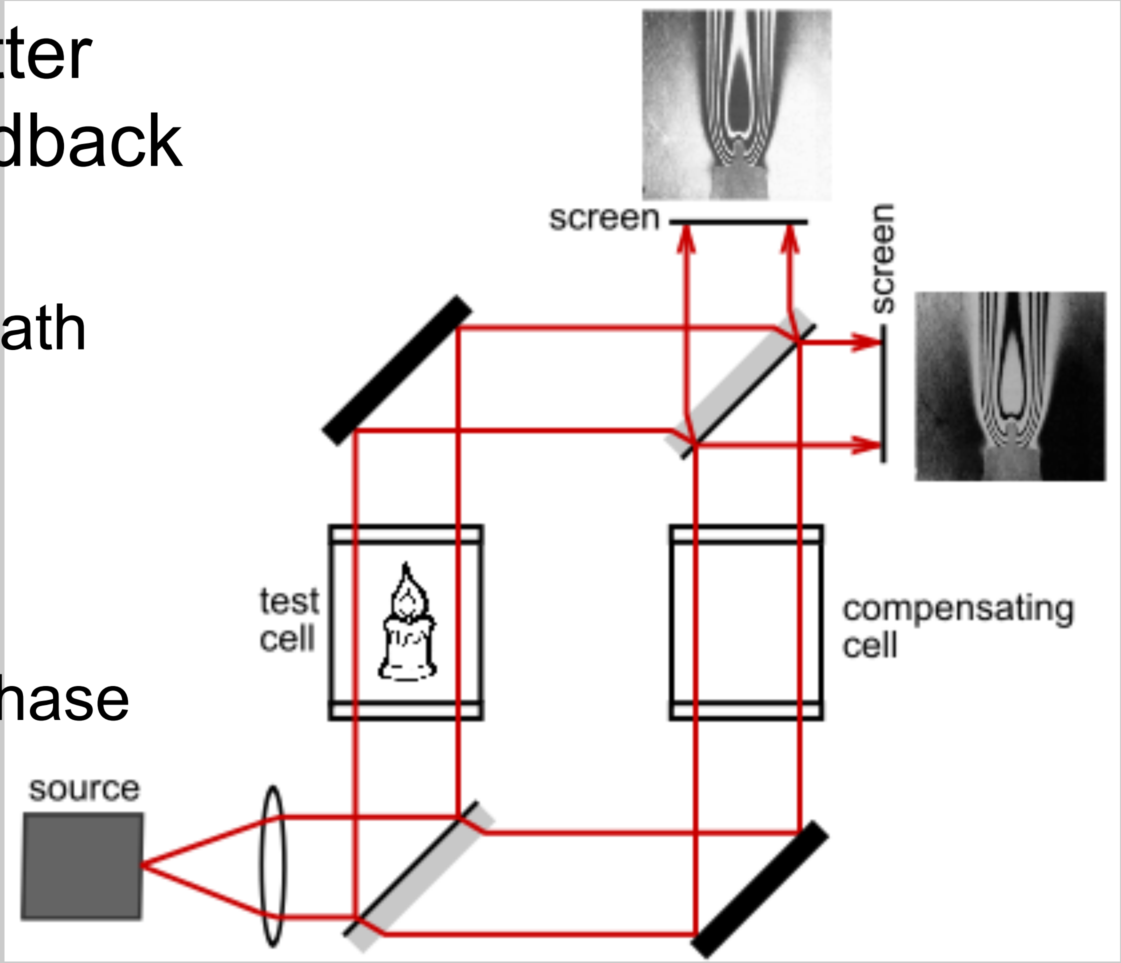
- Measure change in arm length 1000x smaller than a proton (10^{-19} m)
- Fabry-Perot cavity in each arm extends effective length by 280x
- Laser is 200W at 1064nm (Nd:YAG)
- FP plus 'power recycling' mirror enhances intensity 3750x
- Alignment sends most power back towards source:
  – sharpens interference
- Active and passive damping
- Vacuum: 10^{-9} Torr



https://www.ligo.caltech.edu/page/ligos-ifo

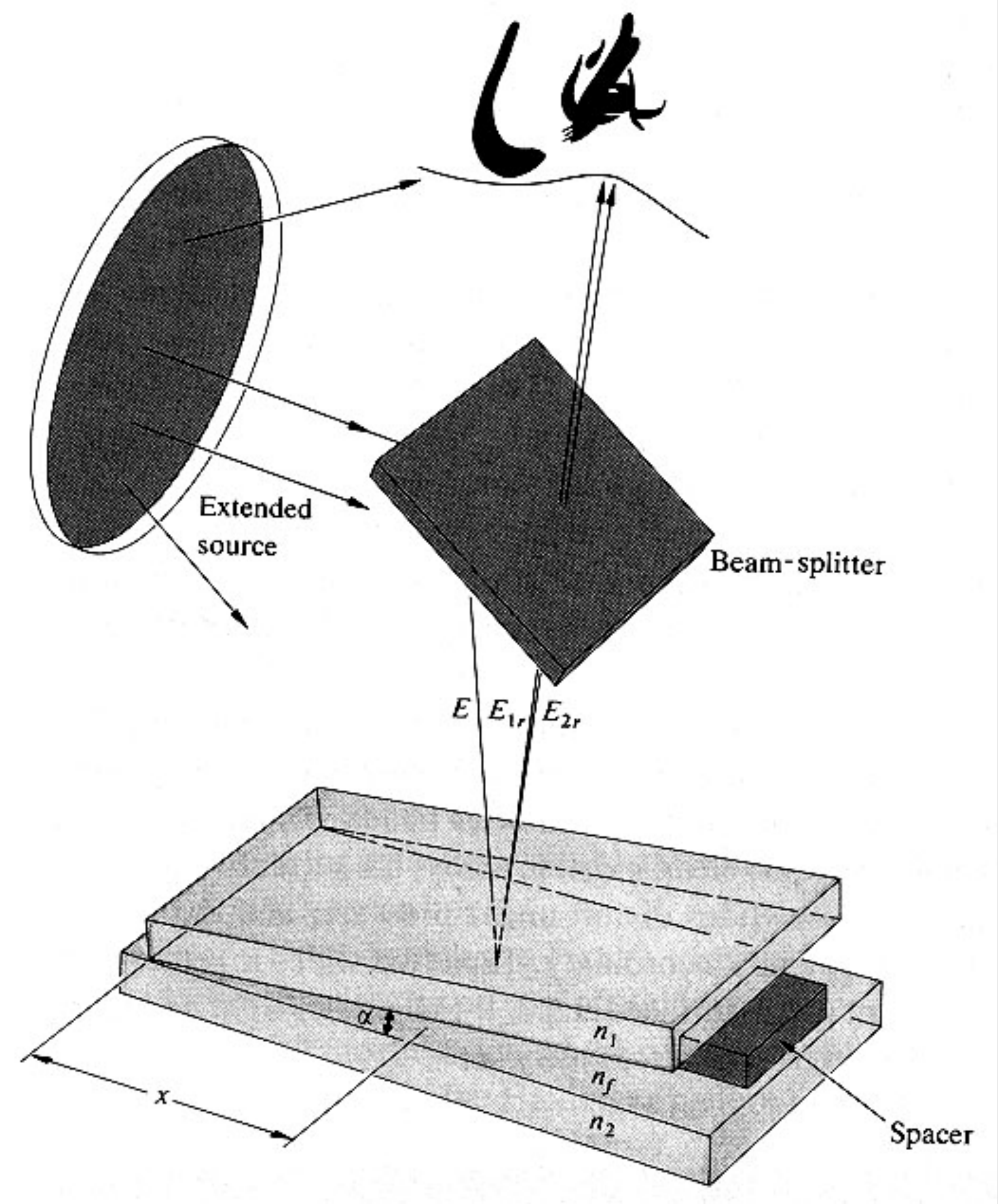
The Mach-Zehnder interferometer

- Double beam-splitter design avoids feedback to source
  - As drawn, equal path
  - Can add legs for adjustable path
- Two output ports
  - Complementary phase



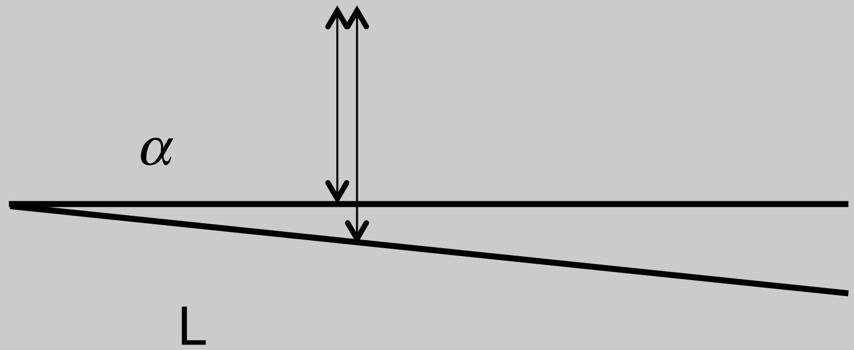
The Fizeau Wedge Interferometer

The Fizeau wedge yields a complex pattern of variable-width fringes, but it can be used to measure the wavelength of a laser beam.



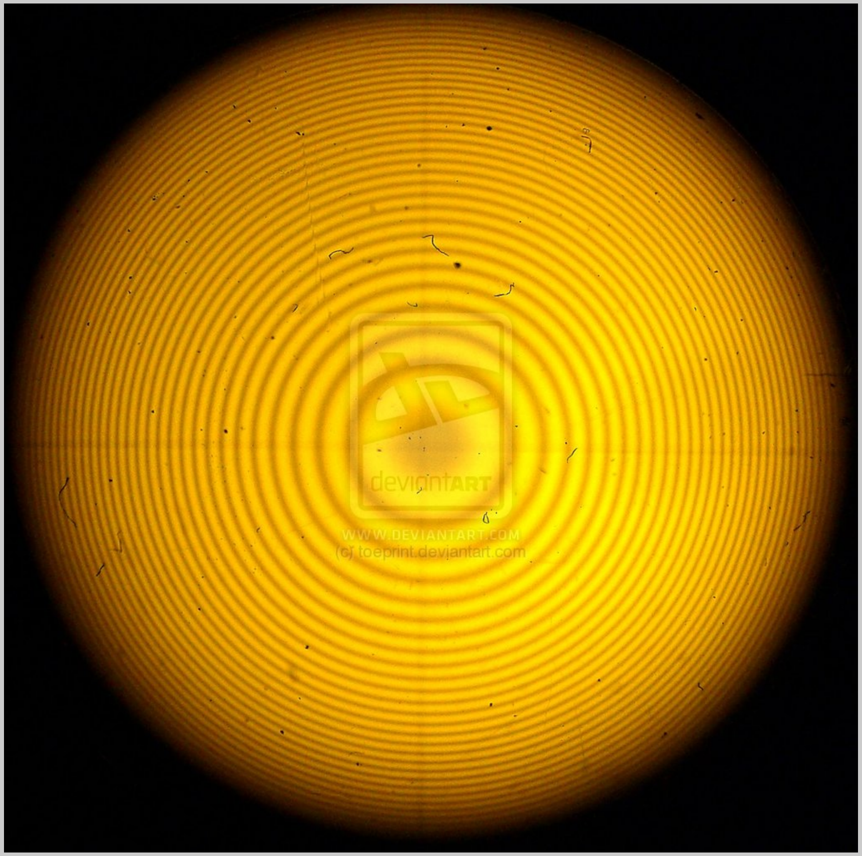
Fizeau wedge calculation

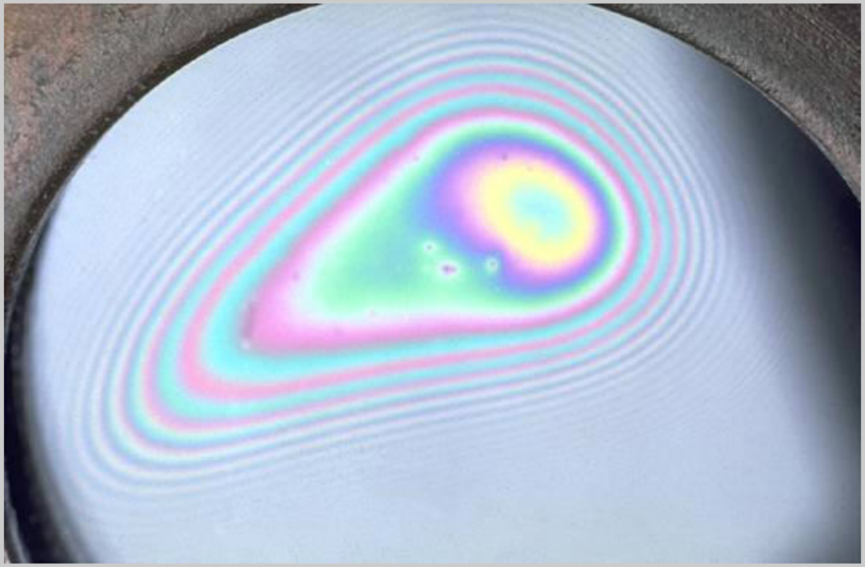
- Interference between reflections from internal surfaces



  - Angle is very small, neglect change in direction
  - Path difference: $\Delta l = 2L\sin\alpha \approx 2L\alpha$
  - Phase difference: $\Delta\phi = \dfrac{\omega}{c} n\,\Delta l \approx 2\pi\dfrac{2L}{\lambda}\alpha$ n = 1 (air gap)
  - Interference: $I_{tot}\left(\Delta\phi\right) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\Delta\phi\right)$
  - One fringe from one max to the next, so maxima are at $\Delta\phi = 2\pi m$
- In this interferometer, minimum path = 0, we can measure absolute wavelength: $\Delta\phi = 2\pi m = 2\pi\dfrac{2L}{\lambda}\alpha \rightarrow \lambda = \dfrac{2L}{m}\alpha$

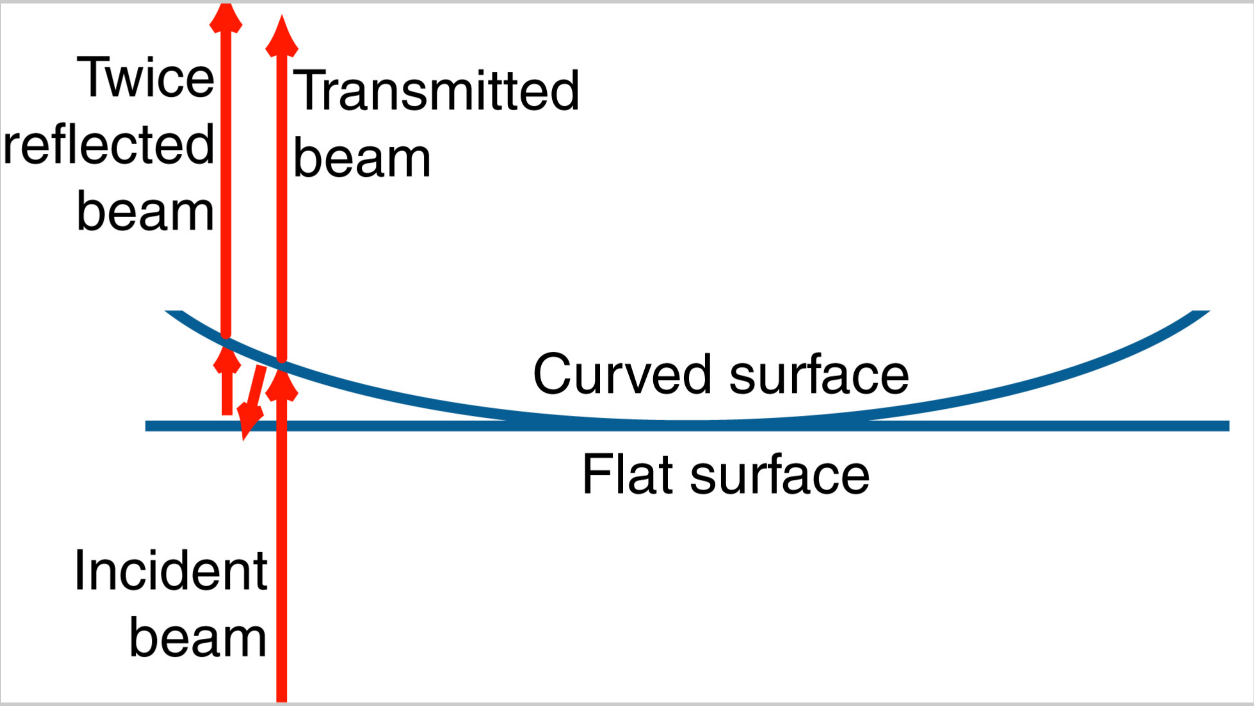
Newton's Rings





Newton's Rings

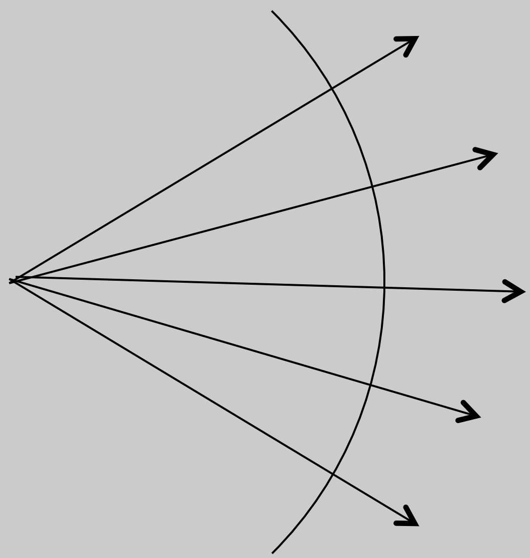
Get constructive interference when an integral number of half wavelengths occur between the two surfaces (that is, when an integral number of full wavelengths occur between the path of the transmitted beam and the twice reflected beam).



This effect also causes the colors in bubbles and oil films on puddles.

Curved wavefronts

- Rays are directed normal to surfaces of constant phase
  - These surfaces are the wavefronts
  - Radius of curvature is approximately at the focal point



- Spherical waves are solutions to the wave equation (away from r = 0)

$$\nabla^2 E + \frac{n^2\omega^2}{c^2} E = 0$$

$$E \propto \frac{1}{r} e^{i(\pm kr - \omega t)}$$

Scalar r
+ outward
- inward

$$I \propto \frac{1}{r^2}$$

Paraxial approximations

- For **rays**, paraxial = small angle to optical axis
  - Ray slope: $\tan\theta \approx \theta$
- For **spherical waves** where power is directed forward:

$$e^{ikr} = \exp\left[ik\sqrt{x^2+y^2+z^2}\right]$$

$$k\sqrt{x^2+y^2+z^2} = kz\sqrt{1+\frac{x^2+y^2}{z^2}} \approx kz\left(1+\frac{x^2+y^2}{2z^2}\right)$$

Expanding to 1st order

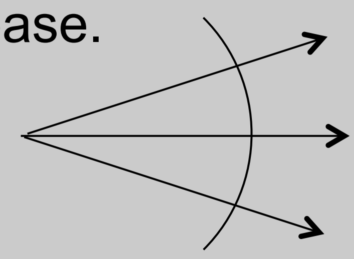
$$e^{i(kr-\omega t)} \to e^{ikz}\exp\left[i\left(k\frac{x^2+y^2}{2z}-\omega t\right)\right]$$

z is radius of curvature = R

Wavefront = surface of constant phase

$$\phi = 0 \to k\frac{x^2+y^2}{2R} = \omega t$$

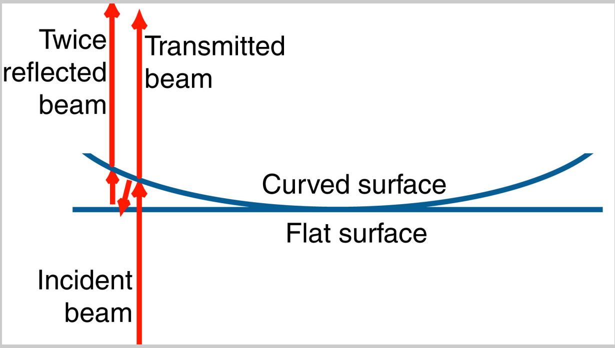
For x, y >0, t must increase.
Wave is diverging:



Newton's rings: interfere plane and spherical waves

- Add two fields:

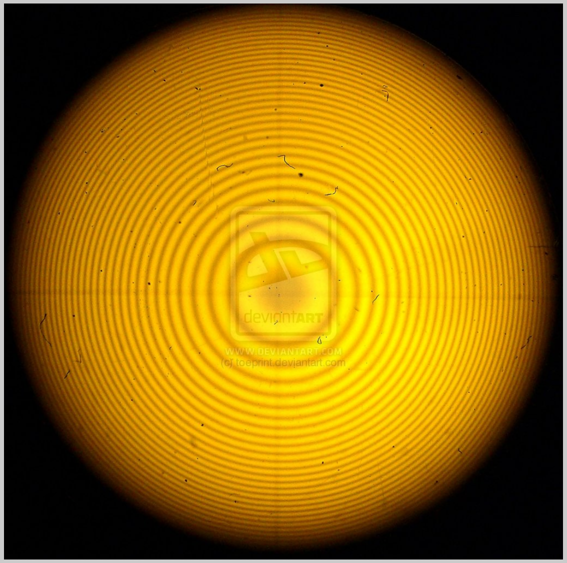
$$E(r) = E_0 + E_0 e^{i\frac{kr^2}{2R}}$$



  - Assume equal amplitude
  - For Newton's rings, 2x phase shift

- Calculate intensity:

$$I(r) \propto \left| E_0 + E_0 e^{i\frac{kr^2}{2R}} \right|^2 = 2E_0^2 + 2E_0^2 \cos\left(\frac{kr^2}{2R}\right)$$

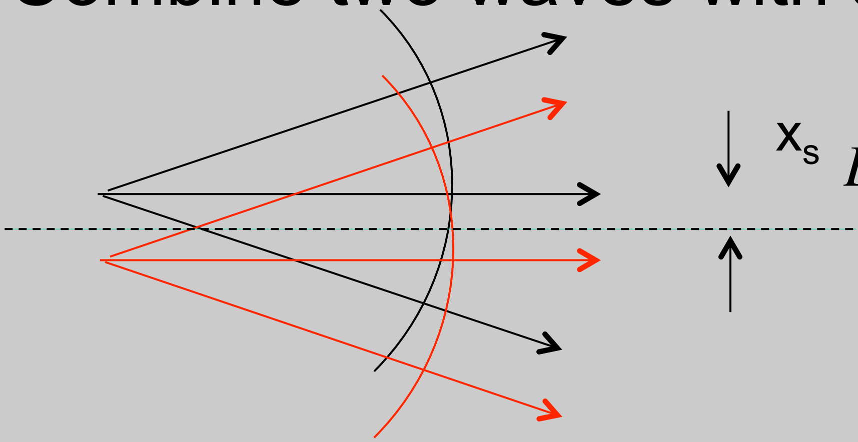


  - Local k increases with r

$$\cos\left(\frac{kr}{2R} r\right) = \cos\left(k_{local} r\right)$$

Shearing interferometer

- Combine two waves with a lateral offset ("shear")



$$E_{tot}(x) = E_0 \exp\left[i \frac{k}{2R}\left((x - x_s)^2 + y^2 \right) \right] + E_0 \exp\left[i \frac{k}{2R}\left((x + x_s)^2 + y^2 \right) \right]$$

$$I_{tot}(x) = I_0 \left(2 + \exp\left[i \left(\frac{k(x - x_s)^2}{2R} - \frac{k(x + x_s)^2}{2R} \right) \right] + c.c. \right)$$

$$(x - x_s)^2 - (x + x_s)^2 = x^2 - 2xx_s + x_s^2 - \left(x^2 + 2xx_s + x_s^2 \right) = 4xx_s$$

$$I_{tot}(x) = 2I_0 \left(1 + \cos\left[\frac{2kx_s}{R} x \right] \right)$$

Fringes are straight, equally spaced
Combine with constant tilt in y direction:
leads to fringe *rotation* with divergence