Homework 9 due 5pm 3 Nov 2006
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Rev 1: 11/1/06

1) HM problem 10-19
2) In this problem, we extend the analysis of problem 1 to show that the refractive index of a molecular medium with a permanent dipole moment is nonlinear with respect to the incident field strength. To make the calculations easier, let's consider a vapor of water rather than the condensed medium of the previous problem. In this case, the incident field E is the same as the effective field E_{mol} at the site of the molecule. The permanent dipole moment for water is 6.2 10^{-30} Coulomb-meter (in SI units).
   a. Let's define the saturation field E_{sat} as the field strength at which the quantity $y = p_0 E_{sat} / kT = 1$. For room temperature, calculate the saturation field strength in V/m and the corresponding saturation time-average intensity in W/m^2. As the incident intensity approaches I_{sat}, nonlinear effects become important.
   b. From the polarization calculated from part a of problem 1 (before expanding as a function of y), calculate the refractive index of water vapor at a pressure of 20 Torr (this is the saturation vapor pressure). Rephrase the expression so that the refractive index is a function of the intensity rather than the field. Plot the refractive index as a function of intensity at room temperature. Make a note of where I_{sat} is on this curve. You should see that the refractive index increases with intensity, and that for low intensity, this function is linear.
   c. Expand the refractive index in a Taylor series to first order in intensity. An intensity-dependent refractive index is often written as $n(I) = n_0 + n_2 I$, where n_0 is the value of the refractive index at low intensity. Calculate the nonlinear coefficient n_2 in m^2/W.
3) HM problem 10-28. The notation for this problem is messy. Starting with Eq. 10.78, plot $\hat{n}_+^2(\omega)$, $\hat{n}_-^2(\omega)$, $\hat{n}^2(\omega)$ for $\omega_c = \frac{3}{2}\omega_p$. This third expression is just the limit where $B \to 0$. By looking at the plot, you can see which frequency ranges are passed (transmitted) or stopped (reflected). Then go to the equations to solve for the characteristic frequencies.

4) HM problem 10-29. Note that this is a case of magnetically induced optical activity (remember the earlier problem on polarization rotation in quartz).
5) Most waveplates are multiple order: they actually give a relative phase delay of many waves. For this problem use the Sellmaier equations for the dispersion of calcite below. Paste these into Mathematica from the pdf. Note that the wavelength, lambda, should be expressed in microns.

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ncalO[lambda_] := Sqrt[(1 + (0.8559 lambda^2)/(lambda^2 - 0.0588^2) +
                  (0.8391 (lambda)^2)/((lambda)^2 - 0.141^2) +
                  (0.0009 (lambda)^2)/((lambda)^2 - 0.197^2) +
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(0.6845 (lambda)^2)/((lambda)^2 - 7.005^2))];

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ncalE[lambda_] := Sqrt[(1 + (1.0856 (lambda)^2)/((lambda)^2 - 0.07897^2) +
                  (0.0988 (lambda)^2)/((lambda)^2 - 0.142^2) +
                  (0.317 (lambda)^2)/((lambda)^2 - 11.468^2))];
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a. Make a plot of ne and no from 400-700 nm.
b. Suppose we want a thickness of approximately 1mm. What is an exact thickness of a multiple-order calcite half-wave plate for a wavelength of 550nm? Assume the optic axis is parallel to the plate surface. How many waves of relative phase delay are there in this design?
c. Because of the chromatic dispersion of calcite, this waveplate will not be exactly half-wave at other wavelengths. Suppose the waveplate is placed with its optic axis at 45^o between crossed polarizers, so that 100% of the intensity at the design wavelength is passed. Plot the intensity transmitted through the second polarizer vs. wavelength. Define the bandwidth as the wavelength range for which the transmission through the second polarizer is above 90%. What is the bandwidth of your waveplate?
d. Now calculate the bandwidth of a “zero-order” half-waveplate. This is a pair of similar waveplates bonded together at 90 degrees. The second plate is cut slightly thinner so that the net phase shift is only one half wave.