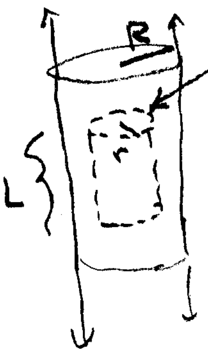
**PHGN361 2010 Exam 1: NAME**

Start from fundamental principles and derive all results. **explain each step for credit.**

1. An infinite cylinder of radius R has charge density $\rho(s) = \alpha/s$ where $s$ is the radial distance from the axis of symmetry. Using the integral form of Gauss's Law to derive an expression for the electric field inside the cylinder.

Fund. Prin: Gauss's Law $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$ Symmetry $\Rightarrow$ $\vec{E}$ radial

$\Rightarrow$ cylindrical Gassian surface inside.




$$\oint \vec{E}\cdot d\vec{a} = \int_{caps} \vec{E}\cdot d\vec{a} + \int_{body} \vec{E}\cdot d\vec{a}$$

$\vec{E} \perp d\vec{a}$ on caps $\Rightarrow \int \vec{E}\cdot d\vec{a} = 0$

$\vec{E} \parallel d\vec{a}$ on body $\Rightarrow \int |\vec{E}||d\vec{a}|$

E same for all tiles

$$\oint \vec{E}\cdot d\vec{a} = E\int_{body} da = E\, 2\pi r L$$

$$Q_{enclosed} = \int \rho\, d\tau = \int_0^L\int_0^r \frac{\alpha}{s}\, 2\pi s\, ds\, dz = 2\pi L r \alpha$$

Put LHS = RHS $\Rightarrow$ $E 2\pi r L = \frac{2\pi r L \alpha}{\epsilon_0}$ $\Rightarrow$ $\vec{E} = \frac{\alpha}{\epsilon_0}\hat{s} + 0\,\hat{\phi} + 0\,\hat{z}$

2. Show that your result obeys the differential form of Gauss's Law. Note $\nabla\cdot\vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Fund. Prin: Gauss's Law $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\alpha}{\epsilon_0}\right) = \frac{1}{s}\frac{\alpha}{\epsilon_0} = \frac{\rho}{\epsilon_0} \Rightarrow \rho = \frac{\alpha}{s} \text{ as stated in (a)}$$