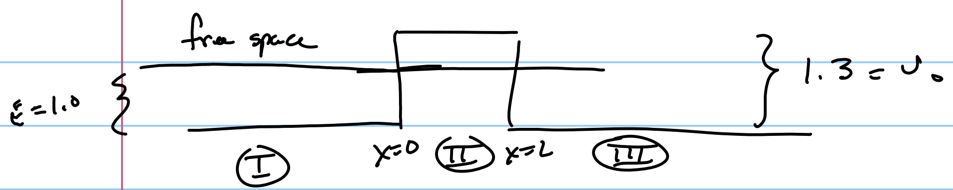
Note Title 7/31/2006

Please return Hwk 3 (Ch 9: 3, 4, 5, 7, 9, 11, 12 ...)



PDE for I $\quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = +i\hbar\frac{\partial \psi}{\partial t}$

Dispersion relation $\quad +\frac{\hbar^2 k^2}{2m} = \cdot i\hbar\, i\omega = \hbar\omega \leftarrow E$

Sep. variables $\quad \psi(x,t) = \varphi(x)\,\Theta(t) \to \hat{a}\, e^{i(kx-\omega t)}$

$$\psi_I = \hat{a}\, e^{i(kx-\omega t)} + \hat{b}\, e^{i(-kx-\omega t)}$$

PDE for II $\quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + U_0\psi = \overset{\hbar\omega}{\overset{\shortparallel}{E}}\psi = i\hbar\frac{\partial}{\partial t}$

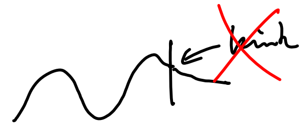
Dispersion relation $\quad \frac{\hbar^2 k^2}{2m} + U_0 = E \qquad k = \sqrt{\frac{2m}{\hbar^2}(E-U_0)} = \alpha$

k is imaginary since $E < U_0$

$$\psi_{II} = \hat{c}\, e^{-\alpha x} e^{-i\omega t} + \hat{d}\, e^{\alpha x} e^{-i\omega t}$$

PDE for III $\quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = i\hbar\frac{\partial}{\partial t}\psi = E\psi$

$$\psi_{III} = \tilde{f}\, e^{i(kx-\omega t)}$$



<u>Bndry cond</u> $\quad \psi_I = \psi_{II} \qquad \left.\frac{\partial \psi_I}{\partial x}\right|_{bndy} = \left.\frac{\partial \psi_{II}}{\partial x}\right|_{bndy}$

$x = 0$

$$\psi_I(x=0) = \psi_{II}(x=0)$$

$$\hat{a}\, e^{i(k0-\cancel{\omega t})} + \hat{b}\, e^{i(k0-\cancel{\omega t})} = \hat{c}\, e^{-\alpha(0)} e^{-i\cancel{\omega t}} + \hat{d}\, e^{\alpha(0)} e^{-i\cancel{\omega t}}$$

$$\boxed{\hat{a} + \hat{b} = \hat{c} + \hat{d}}$$

$$\hat{a}\, ik\, e^{i(k(0)-\cancel{\omega t})} - \hat{b}\, ik\, e^{i(k(0)-\cancel{\omega t})} = -\hat{c}\alpha\, e^{-\alpha(0)} e^{-i\cancel{\omega t}} + \hat{d}\alpha\, e^{\alpha(0)} e^{-i\cancel{\omega t}}$$

$$\boxed{\hat{a}\, ik - \hat{b}\, ik = -\hat{c}\alpha + \hat{d}\alpha}$$

$$\boxed{\hat{c}\, e^{-\alpha L} + \hat{d}\, e^{\alpha L} = \hat{f}\, e^{ikL}}$$

$$\boxed{-\alpha \hat{c}\, e^{-\alpha L} + \hat{d}\alpha\, e^{\alpha L} = ik\, e^{ikL} \hat{f}}$$

4 eqns in $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{f}$

Set $\hat{a} = 1$ 4 eqns in 4 unknowns

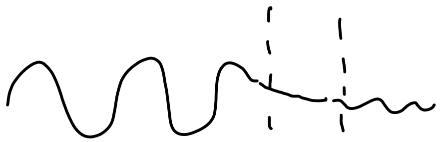
Solve[{ , , , }, { $\hat{b}$, $\hat{c}$, $\hat{d}$, $\hat{f}$ }]

$a \to 1$ $L = 2$ $\hbar = 1$ $m = 1$

$k = \sqrt{\frac{2me}{\hbar^2}}$ energy $\alpha = \sqrt{\frac{2m(u-e)}{\hbar^2}}$

$e = \hbar\omega$ $\omega = \frac{2\pi e}{\hbar}$

$= h\nu$



$t = 0$

I II III

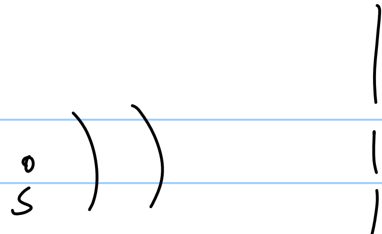
$t = .1$

$t = .2$

```
leftpart[x_, t_] :=
middlepart[x_, t_] :=
rightpart[x_, t_] :=

wavefunction[x_, t_] := If[x<0, leftpart[x,t], If[x<1,
  middlepart[x,t], rightpart[x,t]]]
    Sum[wavefunction[x,t], {e, .6, 1, .05}]
```

Diffraction



find $E(P)$

P

$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ $\qquad$ $\nabla^2 v = \frac{1}{c^2}\frac{\partial^2 v}{\partial t}$

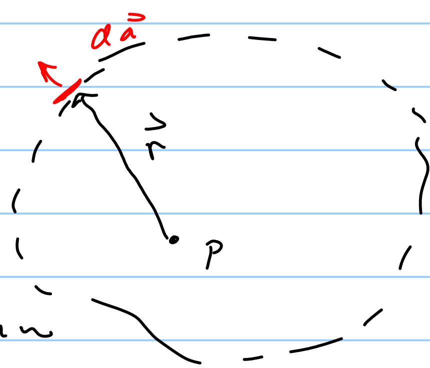
Green's Th. $\qquad$ assume $e^{i\omega t}$ $\qquad$ scalar functions

$$0 = LHS = \int_{vol} \big(\underbrace{V\nabla^2 u - u\nabla^2 V}_{V\frac{1}{c^2}\omega^2 u - u\frac{\omega^2}{c^2}V = 0}\big)\, d\tau = \oint_{surface} \big(V\vec\nabla u - u\vec\nabla V\big)\cdot d\vec a$$

assume

$$V = V_0\,\frac{e^{i(kr+\omega t)}}{r}$$

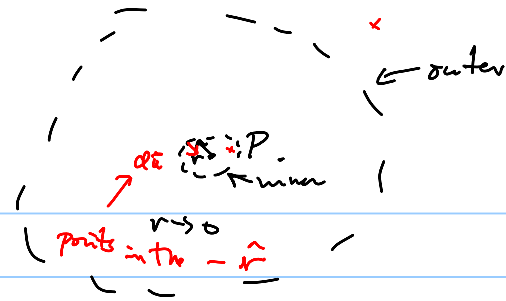
travels to P



$kx-\omega t$ goes out or right

$-kx-\omega t \Leftrightarrow kx+\omega t$ goes left or in

$$\oint_{\text{surface}} \left(v\vec\nabla u - u\vec\nabla v\right)\cdot d\vec a = 0$$



$$\oint_{\substack{\text{outer}\\\text{surface}}} + \oint_{\substack{\text{inner}\\\text{surface}}} = 0$$

↑ spherical shell of radius r

$$\underset{\uparrow\ \text{spherical}}{\vec\nabla} u \cdot d\vec a = -\left(\hat r \frac{\partial}{\partial r} u + \underbrace{\hat\theta \frac{1}{r}\frac{\partial}{\partial\theta} u}_{0} + \underbrace{\hat\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} u}_{0}\right)\cdot \hat r \times r^2 \underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}$$

$$\vec\nabla U(r)\cdot d\vec a = -\underbrace{\left(\frac{\partial U}{\partial r}\right)}_{\text{grad}_n U} r^2\, d\Omega$$

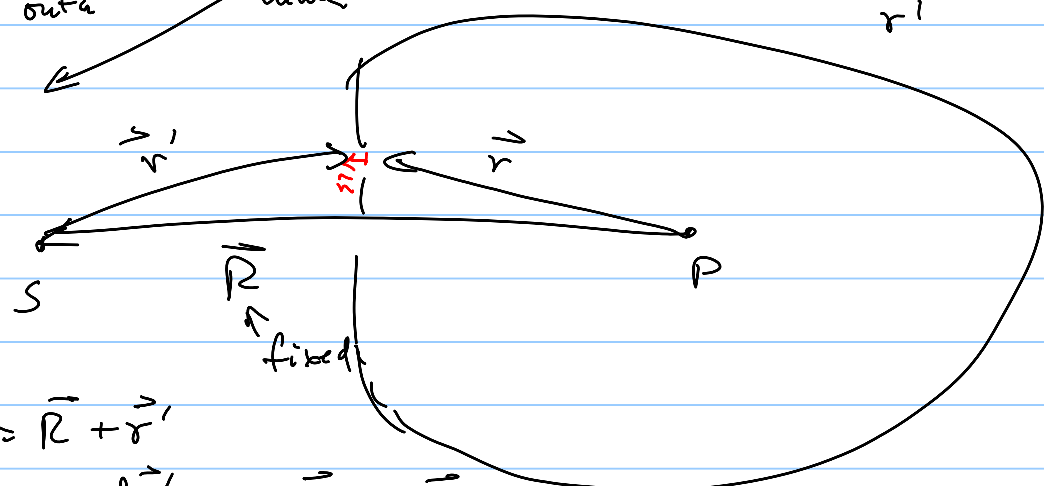
$$\oint_{\text{inner surface}} \left[v\frac{e^{i(kr+\omega t)}}{r}\frac{\partial U}{\partial r} - U\frac{\partial}{\partial r}\left(\frac{v_0 e^{i(kr+\omega t)}}{r}\right)\right] r^2\, d\Omega$$

$$U v_0 \left(\frac{ik}{r} - \frac{1}{r^2}\right) e^{i(kr-\omega t)} \quad r^2\, d\Omega$$

$$\oint_{\text{inner}} U V_0 e^{i(kr+\omega t)}\, d\Omega = \underline{\underline{U(p)}}\, V_0\, e^{i\omega t}\, 4\pi$$

$$\int_{\text{outer}} + \int_{\text{inner}}$$

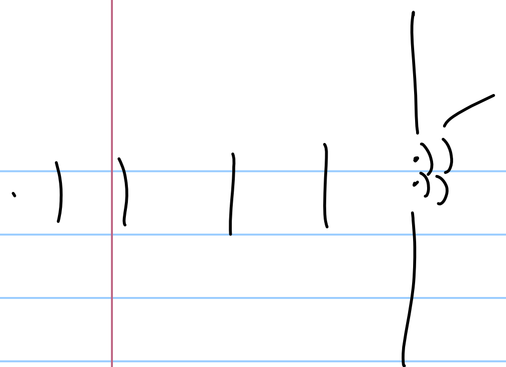
assume $U = U_0 \dfrac{e^{i(kr'-\omega t)}}{r'}$



$\vec{r} = \vec{R} + \vec{r}\,'$

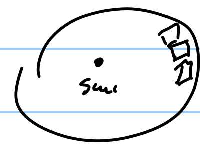
$d\vec{r} = d\vec{r}\,'$ $\qquad \vec{\nabla}_r = \vec{\nabla}_{r'}$

$$U_p = \frac{-ikU_0 e^{-i\omega t}}{4\pi} \int_{\substack{\text{aperture} \\ \text{(outer)}}} \frac{e^{ik(r+r')}}{rr'} \Big[\underbrace{\cos(\hat{n},\vec{r}) - \cos(\hat{n},\vec{r}\,')}_{\text{obliquity factor}} \Big] da$$



$$\frac{e^{i(kr-\omega t)}}{r}$$ spherical wave

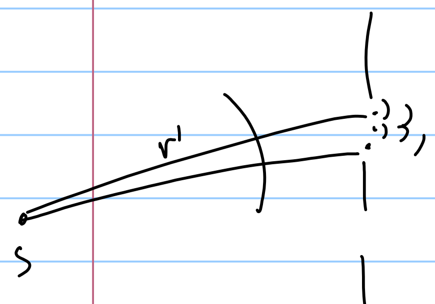
intensity $\propto$ amp^2 $\propto \frac{1}{r^2}$



$$\int \text{intensity} \left(\frac{\text{watts}}{m^2}\right) r^2 \, d\Omega$$

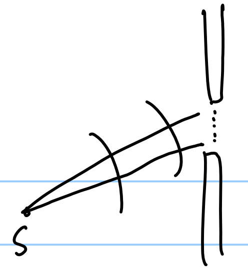
$$\propto \int \frac{1}{r^2} r^2 \, d\Omega = \text{const}$$

Sun 10^9 watts Intensity $\frac{\text{watts}}{m^2}$

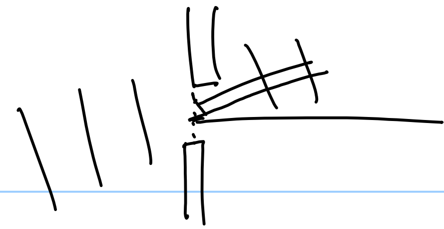


$$\int_{\text{aperture}} e^{ikr'} \frac{e^{i(kr-\omega t)}}{r} \, da$$

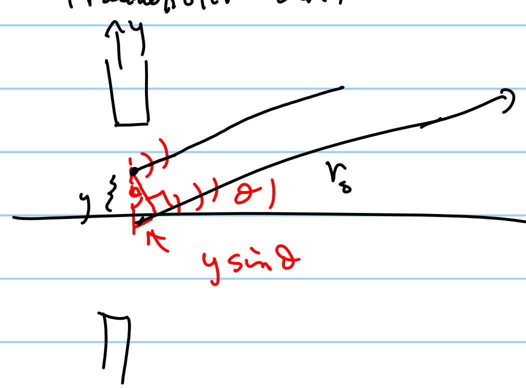
o P



Fresnel diffraction
takes into acount the
curvature of wavefront



Fraunhofer diff



$$\int \_\ e^{ik(r_0 + y\sin\theta)}$$

$da = L\,dy$

$$-e^{ikr_0} L \int_{-w/2}^{w/2} e^{iky\sin\theta}\,dy$$