MATH235 - April 6, 2011 Practice Exam

NAME:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

- 1. (20 Points) For each of the following differential equations list the techniques that can be used to successfully find solutions. The choices are:
 - \mathbf{S} : Separation of Variables
 - **I** : Integrating Factors
 - ${\bf U}\,$: Undetermined Coefficients
 - ${\bf L}\,$: Laplace Transforms
 - \mathbf{E} : Eigenvalues/Eigenvectors

(a)
$$y' = \frac{t^2y - y}{y + 1}$$

(b)
$$y' = \frac{y+t}{t}$$

(c)
$$y' = \cos^2(t)y - \sin(e^{2t})$$

(d)
$$y' = 5y + 6$$

(e)
$$y'' + 9y = \sin(3t)$$

(f)
$$y'' + 9y = \tanh(3t)$$

- (g) $y'' + 9y = \delta_5(t)$
- (h) $y'' + 9y = \sin(t)\cos(t)$
- (i) $\mathbf{Y}' = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$
- (j) $\mathbf{Y}' = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2. (30 Points) Solve each of the following differential equations and, when possible, use initial conditions to solve for any unknown constants.

(a) $y' = t^2 + 2 + (ty)^2 + 2y^2$ where y(0) = 0

(b) $y' + 4t^{-1}y = \cos(3t)$ where y(0) = 0

(c) $y' + 2y = e^{-2t}$

(d) y'' + 4y = 0

(e)
$$y'' + 2y' + 3y = 3e^{-t}\cos(\sqrt{2}t)$$
 where $y(0) = 0$ and $y'(0) = 1$

(f) $2y'' + 8y = t - tu_1(t) - (t - 2)u_1(t) + (t - 2)u_2(t)$ where y(0) = -7 and y'(0) = 3

(g)
$$\mathbf{Y}' = \mathbf{A}\mathbf{Y}$$
 where $\mathbf{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ such that $a, b \in \mathbb{R}$ and $\mathbf{Y}(0) = <1, 0>$