MATH348 - September 30, 2009	**NAME**:
Exam I - 50 Points	**SECTION**:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Short Answer - Justify your response.

   (a) Suppose that $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} \in \mathbb{R}^{n\times n}$, has no solutions. What can be said about solutions to $\mathbf{Ax} = \mathbf{0}$?

   (b) Suppose that $\det(\mathbf{A}) = 0$. Could there exist a solution to $\mathbf{Ax} = \mathbf{b}$ for some $\mathbf{b} \in \mathbb{R}^n$?

   (c) Suppose that the column space of $\mathbf{A}_{n\times n}$ is precisely $\mathbb{R}^n$. What can be said about solutions to $\mathbf{Ax} = \mathbf{0}$?

   (d) Suppose that $\lambda = 0$ is an eigenvalue of $\mathbf{A}$. What can be said about solutions to $\mathbf{Ax} = \mathbf{0}$?

2. (10 Points) Quickies:

(a) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

what is the dimension of the null-space, column-space and row-space of $\mathbf{A}$?

(b) Given,

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right].$$

i. Is inconsistent.

ii. Is consistent with infinitely many solutions.

iii. Is consistent with a unique solution.

(c) Determine if $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$ forms a linearly independent set.

(d) Given,

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 8 & 6 & 4 & 2 \\ 12 & 9 & 6 & 3 \\ 16 & 12 & 8 & 4 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \tag{2}$$

find one eigenvalue of $\mathbf{A}$.

(e) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}, \tag{3}$$

find one eigenvalue of $\mathbf{A}$.

3. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 8 \end{bmatrix}. \tag{4}$$

Find the general solution to $\mathbf{Ax} = \mathbf{b}$.

4. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, \tag{5}$$

Find $\mathbf{A}^{-1}$ and using it solve $\mathbf{Ax} = \mathbf{b} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$.

5. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} -2 & 4 & 0 \\ 0 & 3 & 0 \\ -3 & 6 & 4 \end{bmatrix}. \tag{6}$$

Find all eigenvalues and eigenvectors of $\mathbf{A}$.