Homework 5 due Friday, 11 March 2011

PHGN 570: Physical and Fourier Optics

1. Goodman, problem 2-4. In part (a), you'll end up with a result that is expressed in terms of $g(x, y)$.

2. Goodman, problem 2-14. For the first plot in part c, you can use Plot3D[ ]

3. The propagation of Gaussian beams can be calculated by the method of propagation of the angular spectrum.
   a. Starting with a Gaussian beam at a waist at z = 0,
   $U(\xi,\eta) = U_0 \exp\left[-\left(\frac{\xi^2+\eta^2}{w_0^2}\right)\right]$ calculate the angular spectrum $A(f_x, f_y, z{=}0)$ by taking the Fourier transform. Hint: use Fourier transform pairs or use Mathematica.
   b. Write an expression for the angular spectrum at an arbitrary position $A(f_x, f_y, z)$. Comment on how the power angular spectrum evolves $|A|^2$ in free space.
   c. Do the inverse transform on $A(f_x, f_y, z)$ to obtain the field $U(x,y,z)$.
   Show that $U(x,y,z) = e^{ikz}\frac{1}{1+i\frac{z}{z_0}}\mathrm{Exp}\left[i\left(\frac{k(x^2+y^2)}{2(z-iz_0)}\right)\right]$.
   d. Given that the complex beam parameter q can be expressed as
   $q(z) = (z - iz_0)$, show that $U(x,y,z) = e^{ikz}\frac{q_0}{q(z)}\mathrm{Exp}\left[i\left(\frac{k(x^2+y^2)}{2q(z)}\right)\right]$. From here, this expression can be cast in the more familiar expressions in terms of $w(z)$ and $R(z)$ (but you don't have to do that part).
4. Goodman problem 4-4.
5. Goodman problem 4-9.