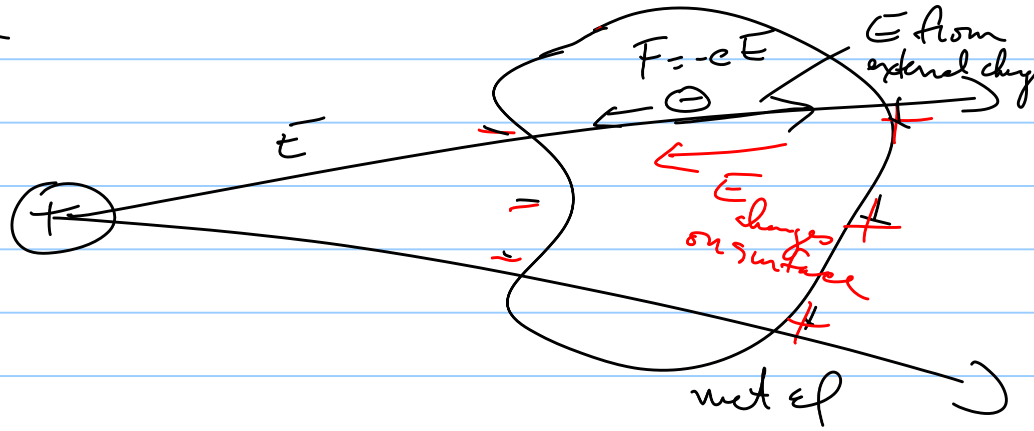
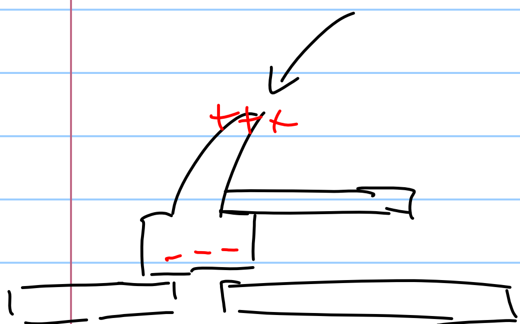
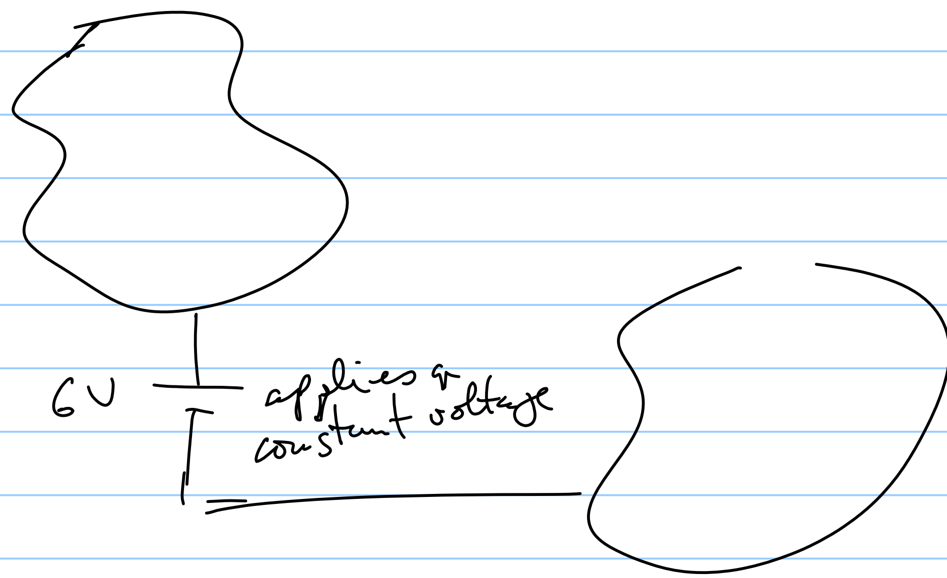
Conductor





some electrons free to move



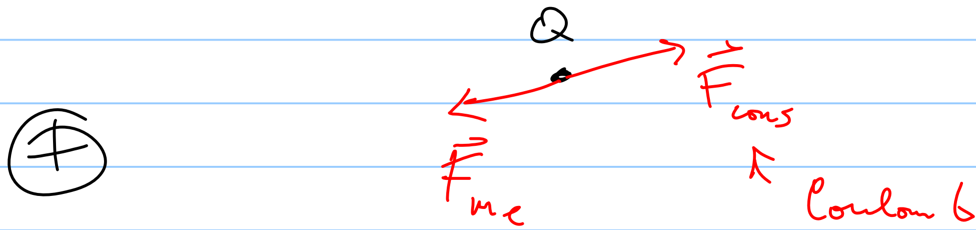
Work energy Theorem

$$W_{tot} = \Delta KE$$

$$W_{non\,cons} + W_{cons} = \Delta KE$$

$$W_{cons} = -\Delta PE$$

$$W_{\text{non con}} = \Delta(KE+PE)$$



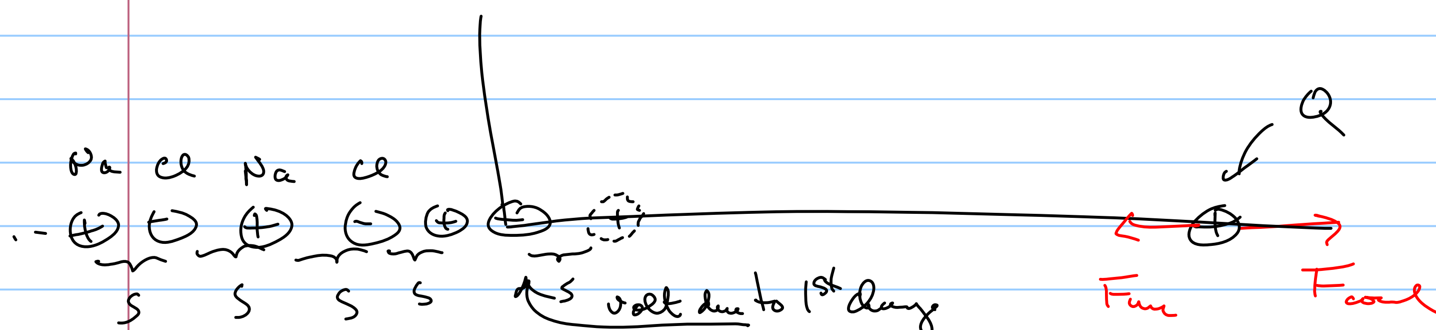
move at const speed $\Delta KE = 0$

$$W_{nc} = W_{me} = -W_{coul} = -(-\Delta PE) = \Delta PE$$

cons $= -\Delta PE$

$$-W_{coul} = -\int Q\vec E\cdot d\vec l = Q\underbrace{\int -\vec E\cdot d\vec l}_{\Delta V} = Q\Delta V = \Delta PE$$

Ex: NaCl crystal in 1-D



$$W_{me} = Q\Delta V = Q\left(\frac{-Q}{4\pi\epsilon_0 s} + \frac{Q}{4\pi\epsilon_0 2s} - \frac{Q}{4\pi\epsilon_0 3s} + \cdots\right)$$

$$= -\frac{Q^2}{4\pi\epsilon_0 s}\left\{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right\} = -\frac{Q^2}{4\pi\epsilon_0 s}\underbrace{\ln(2)}_{.693}$$

$$\ln(1+x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \cdots \qquad -1 < x \le 1$$

How much energy is required to put the crystal together?

$$W = 0$$

$$W_2 = -\frac{Q^2}{4\pi\epsilon_0}\frac{1}{s}$$

$$W_3 = -\frac{Q^2}{4\pi\epsilon_0}\left[-\frac{1}{2s} + \frac{1}{s}\right]$$

$\ominus \quad \oplus \quad \times \quad \leftarrow \ominus$

$\vdots$

$$\sum_{i=1}^{N} W_i \quad \text{energy to assemble crystal}$$

Work required to assemble continuous charge distribution:

Ex: sphere uniformly charge

dr

area $4\pi r^2$ $\qquad d\tau = 4\pi r^2 dr$

$$dW_{me} = dq\, V = dq\, \frac{kq}{r}$$

voltage due to $\boxed{\text{charge already present}}$

$$\int dW_{me} = \int dq\, V = \int_0^R \rho\, 4\pi r^2 dr\, \frac{kq}{r}$$

$\rho \frac{4}{3}\pi r^3$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

R $\qquad$ Q

$$W = \frac{1}{4\pi\epsilon_0} \frac{3}{5} \frac{Q^2}{R}$$