Note Title 11/22/2006

Review

Note: this review occurred before the exam was written.

please see the Lecture notes for 11/27/06 for the areas the 4 questions were drawn from.

Fourier Series vs F. Transf.

applies to functions on a finite interval

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{L}\right) + \sum b_n \sin\left(\frac{n\pi x}{L}\right)$$

a_n, b_n represents discrete freq. content

applies to funct. on an infinite interval

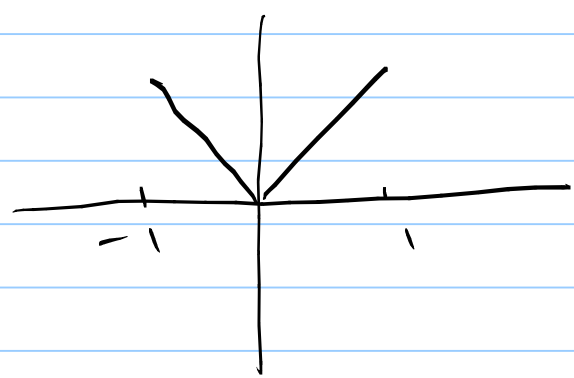
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx} \, dk$$

$F(k)$ represents continuous freq. inf.

Fourier Series always has an interval assoc. with it

e.g.

$[-L, L]$ $\quad$ $[\frac{1}{2}, 1.783]$ eg.



compute the first 2 nonzero terms in the Fourier series for $|x|$ on $[-1, 1]$

example question

Comute the Fourier transform

$f(x) = Ae^{-|x|}$

example question

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A e^{-|x|} e^{ikx} \, dx$$

$$a_0 = \int_{-1}^{1} |x| \, dx = 2 \int_0^1 x \, dx$$

$$a_N = \int_{-1}^{1} \cos(N\pi x/L) \, |x| \, dx$$

$$= \int_0^1 x \cos(N\pi x/L)$$

Same idea applies to any orthog. function expansion e.g.

$$f(x) = \sum_{n=0}^{\infty} c_n P_n(x)$$

$$(P_n, P_m) = \delta_{nm} \left(\frac{2}{2n+1}\right)$$

$P_0 = 1$ compute c_n

$P_1 = x$ for $f(x) = |x|$

$P_2 = \frac{1}{2}(3x^2 - 2)$ on $[-1, 1]$

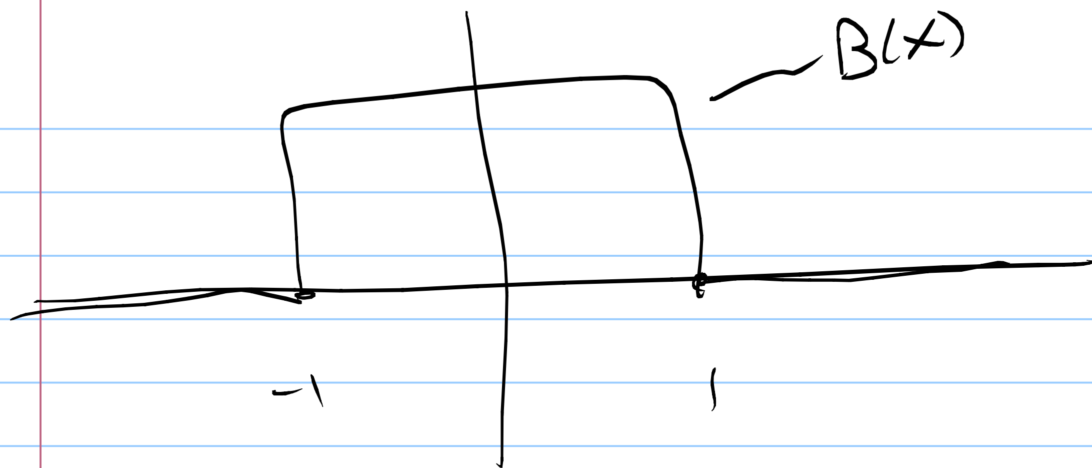
$$|x| = \sum_{n=0}^{\infty} c_n P_n(x)$$

$$(P_\ell, |x|) = \sum_{n=0}^{\infty} c_n \underbrace{(P_\ell, P_n)}_{\frac{2}{2\ell+1}\delta_{\ell,n}}$$

$$= \frac{2}{2\ell+1} c_\ell$$

$$c_\ell = \frac{2\ell+1}{2} (P_\ell, |x|)$$

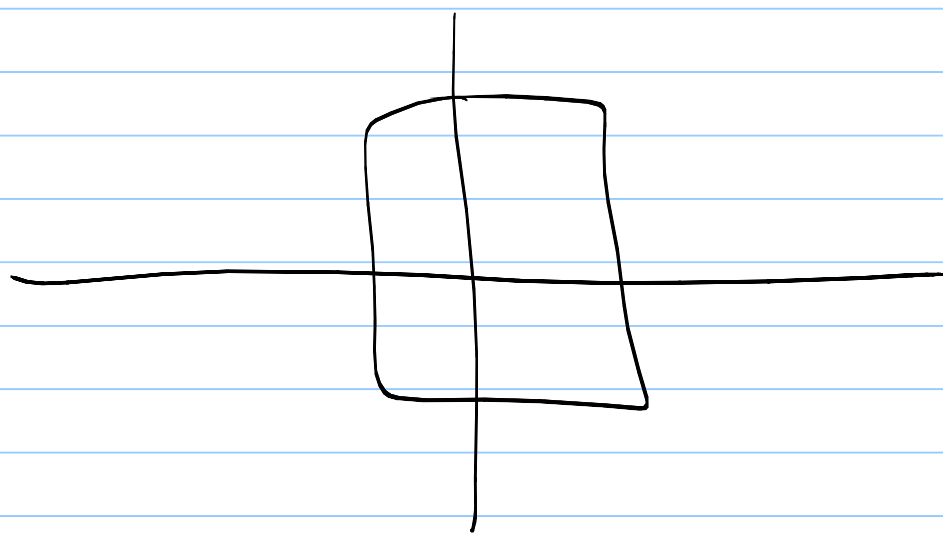
Fourièr transform of a box-car function



You did this

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} B(x)\, e^{ikx}\, dx$$

$$\int_{-1}^{1} e^{ikx}\, dx$$



How about this?

i.e. 2-d Box

$$FT(f(t)) \rightarrow F(\omega)$$

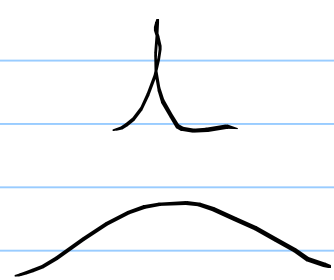
freq. $\sim \frac{1}{\text{time}}$

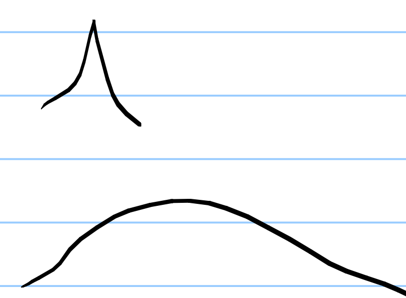
$$FT(f(x)) \rightarrow F(k)$$

wave $\sim \frac{1}{\text{dis}}$

$$e^{-x^2/2\sigma^2} \xrightarrow{FT} \propto e^{-\sigma^2 k^2/2}$$

t ω





a basic idea

know basic properties of
δ - functions

$$\int_I f(x)\,\delta(x-y)\,dx = \begin{cases} f(y) & \text{if } y \in I \\ 0 & \text{otherwise} \end{cases}$$

No Discrete Fourier transform

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we did Sep. of Var. for 1-D wave equation.

Do the same for the diffusion

$$\frac{\partial^2 u(x,t)}{\partial x^2} = K \frac{\partial u(x,t)}{\partial t}$$

Suppose

$$F(\omega) = FT\,(f(t))$$

$|F(\omega)|$ power spectrum

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, dt$$

Rectangular drum

$$\omega_{n,m}^2 = c^2 \pi^2 \left(\frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} \right)$$

if $L_x = L_y = L$

$$\omega_{n,m}^2 = c^2 \pi^2 L^2 \left(n^2 + m^2 \right)$$

$$\Rightarrow \quad \omega_{n,m}^2 = \omega_{m,n}^2 \quad \text{Degeneracy}$$

$$L_x^2 c^2 \pi^2 \left(n^2 + \left(\frac{L_x}{L_y} \right)^2 m^2 \right) = \omega_{n,m}^2$$

understand the general form of solutions to Laplace's equation

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$$\nabla^2 V = 0 \qquad V(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(A_{lm} r^{l} + B_{lm} r^{-(l+1)}\right) Y_{lm}$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + B_l r^{-(l+1)}\right) P_l(\cos\theta)$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$\cancel{a_0} + \cancel{a_1} x + a_2 x^2$$

y' ?

$$2a_2 + 3 \cdot 2 a_3 x$$

$x^2 y''$?

$$2a_2 x^2 + 6 a_3 x^3$$

Review Power Series Solution.