Half Range Expansions - Complex Fourier Series - Applications - Material Summary

1. Given,

$$
f(x)=\left\{\begin{array}{cc}
x, & 0<x \leq L  \tag{1}\\
-x+2 L, & L<x<2 L
\end{array}\right.
$$

(a) Sketch a graph $f$ on $[-4 L, 4 L]$.
(b) Sketch a graph $f^{*}$, the even periodically extended version of $f$, on $[-4 L, 4 L]$.
(c) Calculate the Fourier cosine series for this half-range expansion of $f$.
2. In class we derived the complex Fourier series coefficients $c_{n}$ from the real Fourier series coefficients $a_{0}, a_{n}, b_{n}$. The coefficients $c_{n}$ can also be derived using an orthogonality relation similar to the derivations of $a_{0}, a_{n}, b_{n}$, which is, perhaps, easier.
(a) First show that $\int_{-\pi}^{\pi} e^{i n x} e^{-i m x} d x=2 \pi \delta_{n m}$ where $n, m \in \mathbb{Z}$.
(b) Next using the orthogonality relationship defined in (a), find the Fourier coefficients of $f(x)=\sum_{n=-\infty}^{\infty} c_{n} e^{i n x}$.

Hint: For part (b) you should first multiply both sides of the equation by $e^{-i m x}$. Next you should integrate from $-\pi$ to $\pi$ and use the identity $e^{i k \pi}=(-1)^{k}, \quad k \in \mathbb{Z}$ to derive $c_{m} .{ }^{1}$
Comment: For the complex Fourier series of a $2 L$-periodic function you can make a scaling change of variables like we did for the real Fourier series.
3. Let $f(x)=x^{2},-\pi<x<\pi$, be $2 \pi$-periodic.
(a) Calculate the complex Fourier series representation of $f$.
(b) Using the complex Fourier series found in (a), recover the real Fourier series representation of $f$.

Hint: For part (b) you will want to follow the algorithm discussed in the class.
4. Web-Research - The website, http://grus.berkeley.edu/~jrg/ngst/fft/applicns.html, hosted by Berkeley, contains an informative survey on applications of Fourier transforms. Read one or more of the subsections and:
(a) Briefly summarize the reference in no more than two paragraphs.
(b) List three questions that you have after reading the reference.

Note: The subsections concerning geology and astronomy applies the fast Fourier transform (FFT), which is discussed in the text. We will be discussing this topic when we study matrix algebra. You can read the subsection but it may not make as much sense as the others since we have not yet covered the FFT.
5. Personal Recap - At this point we will pause our work with Fourier series and thus it is a good time to summarize our recent developments. Please do the following for EACH of the sections $11.1,11.2,11.3,11.4$ :
(a) In roughly one paragraph summarize the section. Pretend you are trying to explain what was missed to a friend of yours who missed class. Your goal should be to recount key concepts and connect them to the important mathematics encountered. ${ }^{2}$
(b) List one to three questions remaining with the material.

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[^0]:    ${ }^{1}$ Why is $e^{i k \pi}=(-1)^{k}$ true?
    ${ }^{2}$ Please be concise. The key is to strip the material down to what you feel is essential. If you write too much then you are likely to forget much of the summary.

