MATH348 - March 9, 2012 **NAME:**

Exam I - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points)

   (a) True/False: No Justification Needed

      i. If a periodic function has a jump discontinuity then the graph of its Fourier series will display Gibbs' phenomenon.

      ii. The Fourier transform of a function with no symmetry will have no symmetry.

      iii. If a function has a Fourier series representation then it has a Fourier transform.

      iv. If f is odd and g is even then $\int_{-1}^{2} f(x)g(x)dx = 0$.

      v. The periodic extension of a function is unique.

   (b) Short Response

      i. A Fourier series is the sum of frequency dependent sinusoids, each of which is multiplied by an amplitude of oscillation. Provide a physical interpretation of each of the underlined terms. Also, provide a deficiency/limitation of Fourier series and a way to resolve this issue.

      ii. What is Gibb's phenomenon and when is it expected to occur?

2. (10 Points) Quick Answer Questions

(a) Evaluate $\mathcal{F}\{\delta_{-2}(x) + \delta_2(x)\}$.

(b) Given the following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = \pi,$$

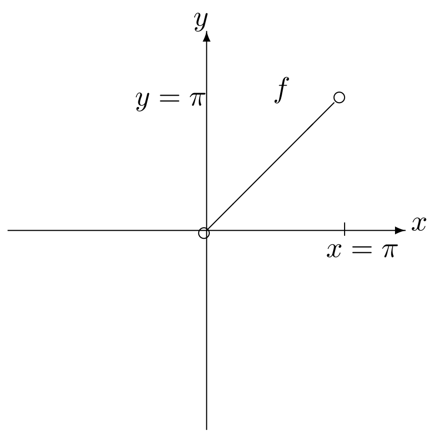
$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \left[\frac{\sin(nx)}{n}\bigg|_{-\pi}^{0} + \frac{\sin(nx)}{n}\bigg|_{0}^{\pi} \right], \quad \int_{-\pi}^{\pi} f(x)\sin(nx)dx = \left[\frac{\cos(nx)}{n}\bigg|_{-\pi}^{0} - \frac{\cos(nx)}{n}\bigg|_{0}^{\pi} \right],$$

$$i\frac{(-1)^n}{n} = \int_{-\pi}^{\pi} g(x)e^{-inx}dx, \quad \frac{e^{in\pi} - e^{-in\pi}}{4\pi} = \int_{-\pi}^{\pi} g(x)dx.$$

fill out the following table using yes/no and justify your choices.

	Is Even	Is Odd
$f(x)$		
$g(x)$		

(c) The graph of f is given below. Label and plot the periodic extension as well as the cosine and sine half-range expansions of f.



3. (10 Points) Find the cosine **and** sine half-range expansions of $f(x) = 1$ for $x \in (0, \pi)$.

4. (10 Points) Find the complex Fourier series representation of $f(x) = x$ for $x \in (-1, 1)$ such that $f(x+2) = f(x)$. Find the complex Fourier series, find the real Fourier series.

5. (10 Points)

(a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & x \in (-1, 1) \\ 0, & x \notin (-1, 1) \end{cases}$.

(b) Evaluate $\mathcal{F}\{f(x+1)\}$.

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1. (10 Points)

   (a) True/False: No Justification Needed

   i. A truncated Fourier sine half-range expansion of a continuous function can have Gibb's phenomenon.

   ii. If a function is even then its Fourier transform will be odd.

   iii. If a function has a Fourier transform then it has a Fourier series.

   iv. If f is odd and g is odd then $\int_{-2}^{2} f(x)g(x)dx = 0$.

   v. A function can have many periodic extensions.

   (b) Short Response

   i. A Fourier series is the sum of frequency dependent sinusoids, each of which is multiplied by an amplitude of oscillation. Provide a physical interpretation of each of the underlined terms. Also, provide a deficiency/limitation of Fourier series and a way to resolve this issue.

   ii. What is a periodic extension, when and why can it be used?

2. (10 Points) Quick Answer Questions

(a) Evaluate $\mathcal{F}\{\delta_2(x) - \delta_{-2}(x)\}$.

(b) Given the following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = \pi,$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \left[\frac{\sin(nx)}{n}\Bigg|_{-\pi}^{0} + \frac{\sin(nx)}{n}\Bigg|_{0}^{\pi}\right], \int_{-\pi}^{\pi} f(x)\sin(nx)dx = \left[\frac{\cos(nx)}{n}\Bigg|_{-\pi}^{0} - \frac{\cos(nx)}{n}\Bigg|_{0}^{\pi}\right],$$

$$i\frac{(-1)^n}{n} = \int_{-\pi}^{\pi} g(x)e^{-inx}dx, \quad \frac{e^{in\pi} - e^{-in\pi}}{4\pi} = \int_{-\pi}^{\pi} g(x)dx.$$

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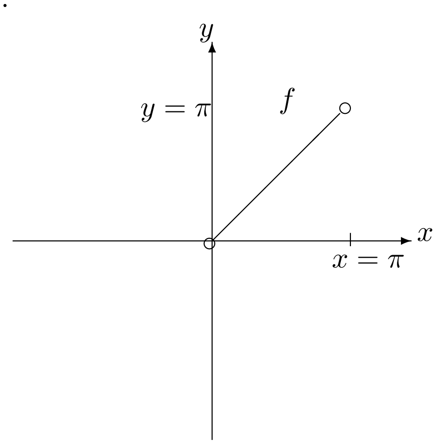

3. (10 Points) Find the cosine **and** sine half-range expansions of $f(x) = 1$ for $x \in (0, \pi)$.

4. (10 Points) Find the complex Fourier series representation of $f(x) = 1 - |x|$ for $x \in (-1, 1)$ such that $f(x + 2) = f(x)$. From the complex Fourier series, find the real Fourier series.

5. (10 Points)

(a) Find the Fourier transform of $f(x) = \begin{cases} 1, & x \in [0,1) \\ -1, & x \in (-1,0) \\ 0, & x \notin (-1,1) \end{cases}$

(b) Evaluate $\mathcal{F}\{f(3x)\}$.

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1. (10 Points)

   (a) True/False: No Justification Needed

   i. If a periodic function is neither even nor odd then its Fourier series representation must have sine terms/modes.

   ii. The Fourier transform of an odd function is odd.

   iii. There are real Fourier series that cannot be represented with complex Fourier series.

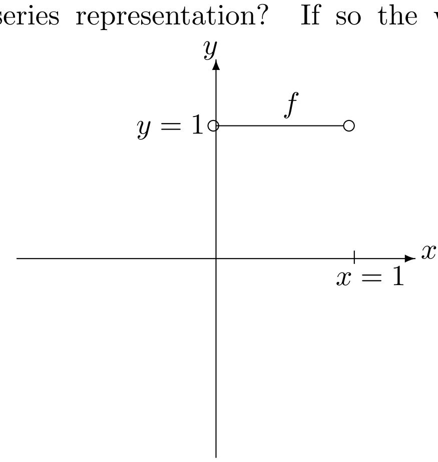
   iv. If f is odd and g is even then $\int_{-1}^{2} f(x)g(x)dx = 0$.

   v. The function e^{ix} has odd symmetry.

   (b) Short Response

   i. A Fourier series is the sum of frequency dependent sinusoids, each of which is multiplied by an amplitude of oscillation. Provide a physical interpretation of each of the underlined terms. Also, provide a deficiency/limitation of Fourier series and a way to resolve this issue.

   ii. Does f have a Fourier series representation? If so the will it contain any cosine functions?



2. (10 Points) Quick Answer Questions

(a) Evaluate $\mathcal{F}\left\{\delta_{-2}(x) - \delta_2(x)\right\}$.

(b) Given the following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = \pi,$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \left[\frac{\sin(nx)}{n}\bigg|_{-\pi}^{0} + \frac{\sin(nx)}{n}\bigg|_{0}^{\pi}\right], \quad \int_{-\pi}^{\pi} f(x)\sin(nx)dx = \left[\frac{\cos(nx)}{n}\bigg|_{-\pi}^{0} - \frac{\cos(nx)}{n}\bigg|_{0}^{\pi}\right],$$

$$i\frac{(-1)^n}{n} = \int_{-\pi}^{\pi} g(x)e^{-inx}dx, \quad \frac{e^{in\pi} - e^{-in\pi}}{4\pi} = \int_{-\pi}^{\pi} g(x)dx.$$

fill out the following table using yes/no and justify your choices.

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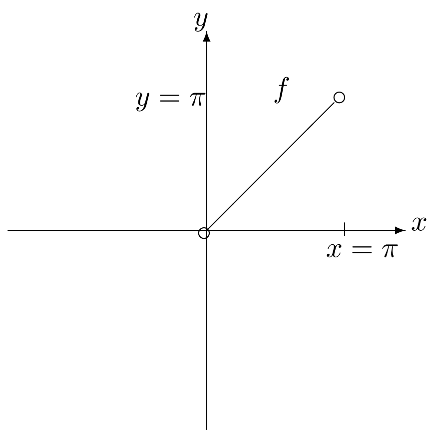

3. (10 Points) Find the cosine **and** sine half-range expansions of $f(x) = x$ for $x \in (0, \pi)$.

4. (10 Points) Find the complex Fourier series representation of $f(x) = \begin{cases} 1, & x \in (0,1) \\ -1, & x \in (-1,0) \end{cases}$, $f(x+2) = f(x)$.
From the complex Fourier series, find the real Fourier series.

5. (10 Points)

(a) Find the Fourier transform of $f(x) = \begin{cases} x, & x \in (-\pi, \pi) \\ 0, & x \notin (-\pi, \pi) \end{cases}$

(b) Assuming that $\hat{f}$ exists, evaluate $\mathcal{F}\{f'(x)\}$.