Chapter 10 & 11

Note Title 6/27/2006

Potential formulation of M.E. (V & $\vec{A}$)

$\vec{B} = \vec{\nabla} \times \vec{A}$ defn of $\vec{A}$

$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$ another ME $\vec{\nabla} \cdot \vec{B} = 0$

for any vector function

How do both $\vec{B}$ & $\vec{E}$ depend on V & $\vec{A}$

(1) $\vec{\nabla} V$

(2) $\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

(3) $\frac{\partial \vec{A}}{\partial t}$

(4) $-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

(5) none

$\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial\, \vec{\nabla} \times \vec{A}}{\partial t} = \vec{\nabla} \times \left(-\frac{\partial A}{\partial t}\right)$$

$$\vec{\nabla} \times \underbrace{\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right)}_{-\vec{\nabla} V} = 0 \qquad \vec{\nabla} \times \vec{G} = 0$$

$$\vec{G} = \vec{\nabla} \chi$$

V ↖ voltage or scalar potential

Faradays

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} V \quad \text{or} \quad \boxed{\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}}$$

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$

fields in terms of potential

Instead of ME we can PDE in terms of $\vec{A}$ & V

Amps law

I

$$\vec{\nabla} \times \vec{B} = \vec{\nabla} \times \vec{\nabla} \times \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} - \mu_0 \epsilon_0 \vec{\nabla} \frac{\partial V}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

Looks like a mess. Rearange

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

II

$$\left(\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}\right) - \vec{\nabla}\left(\underline{\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}}\right) = -\mu_0 \vec{J}$$

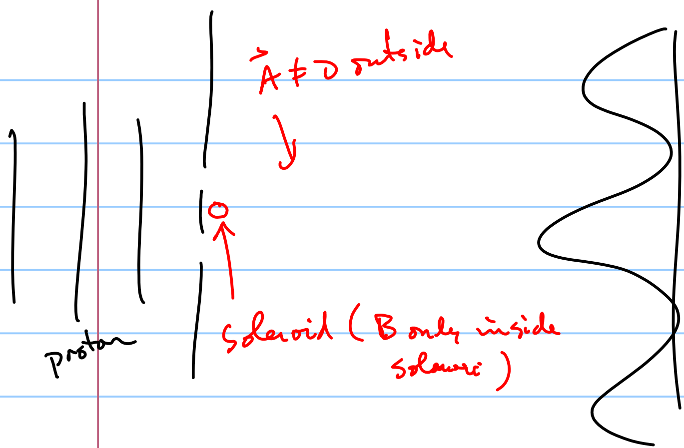
1

Need to know $\vec{A}$ & V (4 functions rather than $\vec{E}$ & $\vec{B}$, 6)

In Quantum it is $\vec{A}$ & V that go into Sch. eqn

$$PE = qV$$

$$\vec{p} \to \left(\frac{\hbar}{i}\vec{\nabla} - q\vec{A}\right)$$



$\vec{A}$ & V are not unique: Gauge trans which don't affect $\vec{E}$ or $\vec{B}$

scalar function

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda \qquad V' = V - \frac{\partial \lambda(x,y,z,t)}{\partial t}$$

$$\vec{B} = \vec{\nabla}\times(\vec{A} + \vec{\nabla}\lambda) = B'$$

$$= \vec{\nabla}\times\vec{A} + \underbrace{\vec{\nabla}\times\vec{\nabla}\lambda}_{0}$$

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E}' = \vec{E}$$

Last ME in terms of $\vec{A}, V$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla}\cdot\left(-\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}\right) = \frac{\rho}{\epsilon_0}$$

$$\mathrm{I} \qquad \nabla^2 V + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\frac{\rho}{\epsilon_0}$$

2 choices for gauge

1.) Coulomb gauge: $\vec{\nabla}\cdot\vec{A} = 0$

$$\vec{\nabla}\cdot(\vec{A} + \vec{\nabla}\lambda) = \vec{\nabla}\cdot\vec{A} + \underbrace{\vec{\nabla}\cdot\vec{\nabla}\lambda}_{\text{choose } \lambda \ni \text{lhs} = 0} = 0$$

$$\mathrm{I} \qquad \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\mathrm{II} \qquad \nabla^2\vec{A} - \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = \mu_0\vec{J} + \mu_0\epsilon_0\vec{\nabla}\frac{\partial V}{\partial t}$$

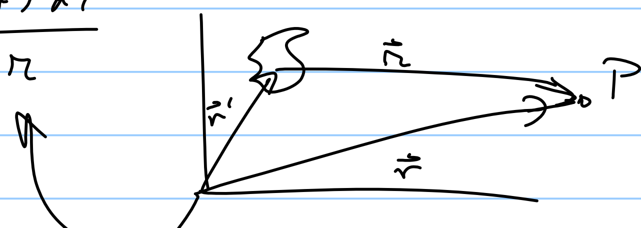
2.) Lorentz gauge $\qquad \vec{\nabla}\cdot\vec{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0} \qquad \nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

time indep solns: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ Poisson's eqn

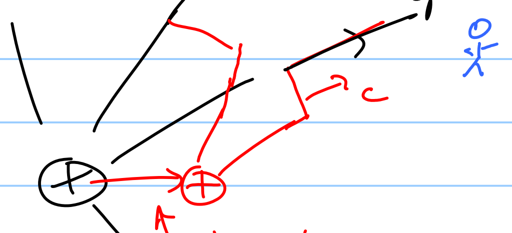
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{r} \qquad \int \frac{k\,dQ}{r} \quad Q \quad V \quad E$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\,d\tau'}{r}$$

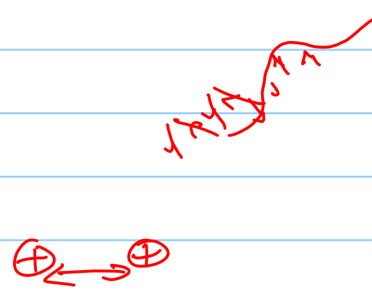


$$|\vec{r}| = |\vec{r} - \vec{r}\,'| =$$

EM wave travels at speed c

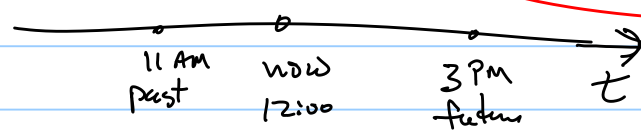


step motion



sinusoidal motion

retarded time: t now $\quad t_{retarded} = \underset{\uparrow \text{now}}{t} - \frac{\mathscr{r}}{c}$

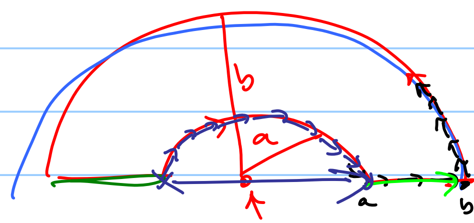


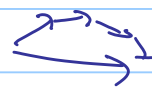
might guess that we need only ~~find~~ $\vec{E}$ & $\vec{B}$ at retarded times

We need to use $\vec{A}$ & V with ρ, J evaluated at retarded times $\Rightarrow$ what is measured

$$\underset{\uparrow \text{now}}{\vec{A}(\vec{r},t)} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}} \qquad V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)\,d\tau'}{\mathscr{r}}$$

10.10





$I(t) = kt$

$(t - \frac{\mathscr{r}}{c})$

$\vec{J}\,d\tau \longrightarrow I\,d\vec{l}$

$$\int \frac{k t_r\,d\vec{l}}{\mathscr{r}} = \int \frac{k(t - \frac{\mathscr{r}}{c})\,d\vec{l}}{\mathscr{r}}$$

$$= \int \frac{kt}{r}\, d\vec{\ell} - \int k \frac{\vec{r}}{\cancel{r}}\, d\vec{\ell} = kt \int \frac{d\vec{\ell}}{r} - \cancel{\frac{k}{c} \int d\vec{\ell}} \to 0$$

$$\int_a^b \frac{dx\,\hat{x}}{|x|} + \int_\pi^0 \frac{b\,d\theta\,\hat{\theta}}{b} + \int_{-b}^{-a} \frac{dx\,\hat{x}}{|x|} + \int \frac{a\,d\theta\,\hat{\theta}}{a}$$