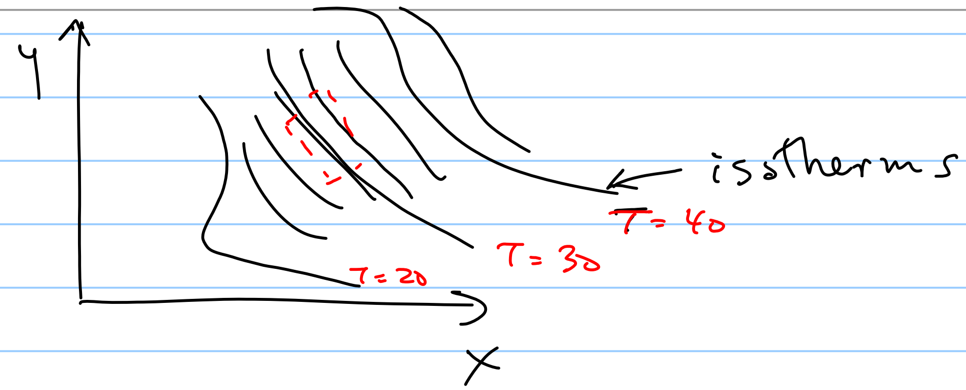
Vector calculus in thermo

Note Title 2/6/2008



T(x, y) scalar field. Consider flow of heat

- has direction (doesn't flow along isotherm)
- magnitude $\dfrac{\text{thermal energy}}{\text{area} \cdot \text{time}}$ $\vec{h}$ heat flow vector

current density $\vec{j} = \rho \vec{v}$ ← velocity $\dfrac{\text{charge}}{m^2 \cdot s}$

charge density

How are $\vec{h}$ & T related in a slab (area A thickness d)

heat flow $\propto$ area of faces

$\propto$ temp difference between faces

$\propto \dfrac{1}{\text{distance between plates}}$

$\propto$ material const (thermal conductivity)

$$J = K(T_2 - T_1)\frac{A}{d}$$

$\dfrac{\text{thermal energy}}{\text{time}}$

for infinitesimal slab $\Delta J = K \Delta T \frac{\Delta A}{\Delta S}$

$$|\vec{h}| = \frac{\Delta J}{\Delta A} = K \frac{\Delta T}{\Delta S}$$

gradient

$$\boxed{\underline{\vec{h} = -K \vec{\nabla} T}}$$

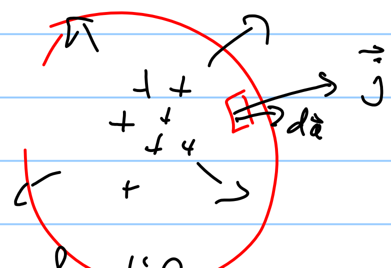
Not a fundamental law. It is material dep. (like Ohm's Law)

points uphill

heat flows from higher to lower temperature

charge flow

$\rho \left(\frac{\text{Coul}}{m^2 \cdot s}\right)$



$$\vec{j} = \rho \vec{v} \left(\frac{\text{Coul}}{m^2 \cdot s}\right)$$

$\underbrace{\vec{j} \cdot d\vec{a}}$ flux through tile

$\frac{\text{Coul}}{m^2 \cdot s} \, m^2 \to \frac{\text{Coul}}{s}$

$$\oint \vec{j} \cdot d\vec{a} = -\frac{d}{dt} Q_{encl}$$

$\downarrow$ div Th.

$\rho(x, y, z, t)$

$$\int \vec{\nabla} \cdot \vec{j} \, d\tau = -\frac{d}{dt} \int \rho \, d\tau = \int \left(-\frac{\partial \rho}{\partial t}\right) d\tau$$

$$\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t} \qquad \text{cons. charge}$$

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heat flux

$$\oint \vec{h} \cdot d\vec{a} = -\frac{d}{dt} G_{encl.}$$

$\vec{h}$: $\frac{\text{thermal energy}}{m^2 \cdot s}$

$G_{encl.}$: Thermal energy enclosed by surface

$\downarrow$

$$\int \vec{\nabla} \cdot \vec{h} \, d\tau = -\frac{d}{dt} \int g \, d\tau$$

g: thermal energy density

$$\vec\nabla \cdot \vec h = -\frac{\partial}{\partial t} g$$

cons. of heat energy

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$$-K \vec\nabla T$$

$$-K \nabla^2 T = -\frac{\partial g}{\partial t}$$

g thermal energy density

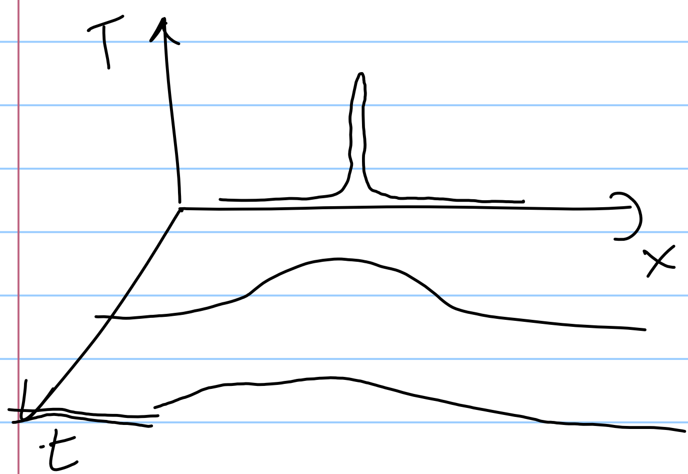
$\propto$ Temp

$$\boxed{-K \nabla^2 T \propto -\frac{\partial T}{\partial t}}$$

Schrod eqn free particle
(not complex

1-D copper

$$\nabla^2 T \xrightarrow{1\text{-}D} \frac{\partial^2 T}{\partial x^2}$$



Thermal statics

$$\nabla^2 T = -\frac{\partial T}{\partial t} \longrightarrow 0$$

Laplaces eqn!