Reading: Today : 12.2

Tomorrow: 12.3

Last time: covarient / contravarient vectors,

$$x^{\mu} = (ct, x, y, z)$$

$$x^0 \quad x^1 \quad x^2 \quad x^3$$

Contravarient $\quad \bar{x}^{\nu} = \frac{\partial \bar{x}^{\nu}}{\partial x^{\mu}} x^{\mu}$

any old contravarient vector

$$\bar{a}^{\nu} = \frac{\partial \bar{x}^{\nu}}{\partial x^{\mu}} a^{\mu}$$

for covarient vectors

$$\bar{a}_{\nu} = \frac{\partial x^{\mu}}{\partial \bar{x}^{\nu}} a_{\mu}$$

Use Lorentz transform for $\bar{S}$ going vel. v w respect to S in x-direction

$\bar{x} = \gamma(x - vt)$

$\bar{y} = y$

$\bar{z} = z$

$\bar{t} = \gamma(t - \frac{v}{c^2}x)$

Find $\frac{\partial \bar{x}^\nu}{\partial x^\mu}$ matrix

In this case: $\frac{\partial \bar{x}^\nu}{\partial x^\mu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$\bar{x}^\nu = \frac{\partial \bar{x}^\nu}{\partial x^\mu} x^\mu$$

If this guy were $\Lambda_{\mu\nu}$

$\rightarrow \Lambda_{\mu\nu} x^\mu = \bar{x}_\nu$

So what about the operator $\frac{\partial}{\partial x^\mu}$?

$\partial \bar{x}^\nu = \bar{x}_1^\nu - \bar{x}_0^\nu \Rightarrow \frac{\partial}{\partial x^\mu}$ is covarient

A lot of times you'll see people write

$$\frac{\partial}{\partial x^\mu} = \partial_\mu = \frac{\partial}{\partial x^0}\hat{x}^0 + \frac{\partial}{\partial x^1}\hat{x}^1 + \frac{\partial}{\partial x^2}\hat{x}^2 + \frac{\partial}{\partial x^3}\hat{x}^3$$

This operator replaces $\vec{\nabla}$ in 3D.

Find $\partial_\mu \partial^\mu$: $\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

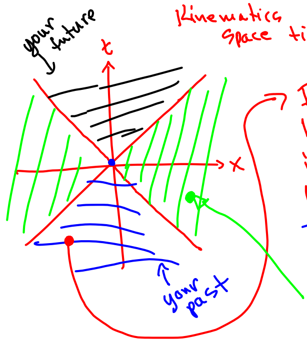
$\partial^\mu = g_{\mu\nu}\partial^\nu = \left(\frac{-1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\partial_\mu \partial^\mu = g_{\mu\nu}\partial^\nu \partial^\mu = \underbrace{-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}_{\partial_\mu \partial^\mu}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)V = \frac{-\rho}{\epsilon_0}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\mu_0 \vec{J}$$

Kinematics of Space time



If an event happens here, then it definitely in all references frames happened in my past.

This is the stuff that can affect me

Different observers will say that stuff happened in different orders. Can't effect each other.

Let's use some invarients':

say you have two events. $(ct_1, x_1, y_1, z_1) = x_1^\mu$

$(ct_2, x_2, y_2, z_2) = x_2^\mu$

$$x_1^\mu - x_2^\mu = d^\mu$$

$$= (c\Delta t, \Delta x, \Delta y, \Delta z)$$

$$d_\mu d^\mu = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$I = \underbrace{-c^2\Delta t^2 + d^2}_{\text{The same in all reference frames}}$$

In my ref. if something happens at same time $\rightarrow I = d^2 > \emptyset$, space-like.

For any space-like event, there's a ref. frame where the two events happen at the same time.

In my ref. frame, if something happens at same posn in my past/future, $I = -c\Delta t^2 < 0$ - time-like

For any time-like events, there's a ref. frame wher the two events happen at the same posn.

The boundary is the light line so for two events where $I = \emptyset$, they call that light-like.

Proper time: $d\tau = \sqrt{1-\frac{u^2}{c^2}}\, dt = \frac{1}{\gamma} dt$

Let's say you have a moving particle in your lab moving w/ speed u.

change in time in part. frame

change t for lab frame

Proper velocity: $\vec{\eta} = \gamma \vec{u}$ $\quad \frac{d\vec{l}}{d\tau}$ ← lab frame ← object frame

$\vec{u} = \frac{d\vec{l}}{dt}$

No 4-vector exists for $\vec{u}$. But there is one for $\vec{\eta}$

$$\eta^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{u} \end{pmatrix} \qquad \bar{\eta}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu} \eta^\nu$$

That also means that

$$\eta_\mu \eta^\mu \text{ is invarient}$$

$$-\gamma^2 c^2 + \gamma^2 u^2 = \text{invarient.}$$

If I multiply by $m^2 c^2$

$$-\gamma^2 m^2 c^4 + \gamma^2 m^2 u^2 c^2$$

Next time we'll see that

$$\vec{p} = \gamma m \vec{v} \; ; \; E = \gamma m c^2$$

momentum energy

$$-E^2 + (pc)^2$$