MATH225, Fall 2008 **Name:**
Worksheet 7 (3.6, 4.1, 4.2) **Section:**

For full credit, you must show all work and box answers.

1. Find the solutions of the given second-order equations or initial-value problems. Use the method from section 3.6.
   (a) $y'' + 6y' + 9y = 0$

   (b) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 0$

   (c) $4y'' - 8y' + 3y = 0, \quad y(0) = 2, \quad y'(0) = \dfrac{1}{2}$

2. Consider the harmonic oscillator with the second-order equation $2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = 0$.

   (a) Find the general solution of the second-order equation that models the motion of the oscillator. Use the method from section 3.6.

   (b) Classify the oscillator.

   (c) Find the particular solution with the initial condition $y(0) = 0$ and $v(0) = 3$.

   (d) What is the long-term behavior (as $t \to \infty$) of $y(t)$ and $v(t)$ from part(c)?

   (e) Write the first-order system that corresponds to the second-order differential equation given above, find the eigenvalues, and classify the origin.

3. Find the solutions of the given second-order equations or initial-value problems.

(a) $y'' - 3y' - 4y = 4t^2 - 1$

(b) $\dfrac{d^2y}{dt^2} - 4y = t^2 + 3e^t, \quad y(0) = 0, \quad \left.\dfrac{dy}{dt}\right|_{t=0} = 2$

(c) $y'' - 4y' + 4y = 3e^{2t}$

4. Find the solutions of the given second-order equations or initial-value problems.

(a) $y'' + y = 3\sin(2t), \quad y(0) = y'(0) = 0$

(b) $y'' + 6y' + 5y = 4e^{-t}\cos(3t)$