1.8 Linearity Principles

Linearity Principle:

If $y_h(t)$ is a solution of the homogeneous linear equation:
$\dfrac{dy}{dt} = a(t)y$ or $\frac{dy}{dt} - a(t)y = 0$,
then $ky_h(t)$ is a solution for any constant k

Extended Linearity Principle:

Consider the nonhomogeneous equation:
$\dfrac{dy}{dt} - a(t)y = b(t)$ and its associated homogeneous equation: $\dfrac{dy}{dt} - a(t)y = 0$

1. If $y_h(t)$ is any solution to the homogeneous equation and $y_p(t)$ is any solution to the nonhomogeneous equation, then $y_h(t) + y_p(t)$ is also a solution of the nonhomogeneous equation.
2. Suppose $y_p(t)$ and $y_q(t)$ are two solutions of the nonhomogeneous equation, then $y_p(t) - y_q(t)$ is a solution of the homogenous equation.

Therefore, if $y_h(t)$ is nonzero, $ky_h(t) + y_p(t)$ is the general solution of the nonhomogeneous equation.