Test tomorrow

5 or 6 questions

1/2, 1/2 Ch. 10, Ch. 11.

At least 1/2 from HW.

Lorentz Gauge

$$\vec{\nabla} \cdot \vec{A} = -\epsilon \mu_0 \frac{\partial V}{\partial t}$$

Gauge Transforms

If I give $\vec{A}, V, \lambda$

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda \; ; \; V' = V - \frac{\partial \lambda}{\partial t}$$

Relationship between $\rho, \vec{J}$ and $V, \vec{A}$

Lorentz Gauge

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)V = \frac{-\rho}{\epsilon_0} \; ; \; \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\mu_0 \vec{J}$$

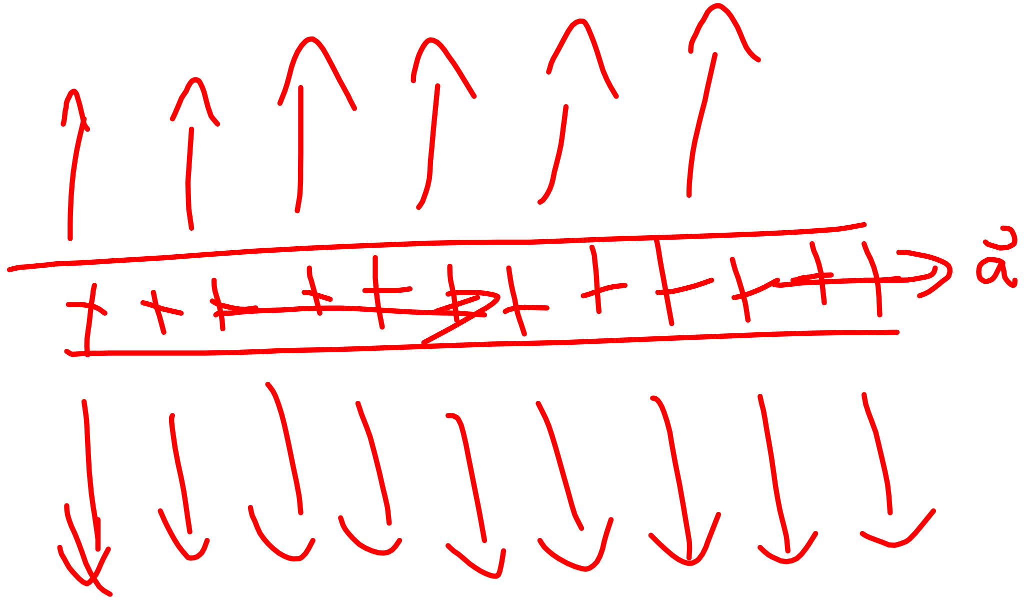
$$V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r}\,', t_r)}{\eta}\, d\tau'$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}\,', t_r)}{\eta}\, d\tau'$$

Potent. $\Longleftrightarrow$ Field.

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \left[\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}^2}\hat{\mathscr{r}} + \frac{\dot{\rho}(\vec{r}\,',t_r)}{c\mathscr{r}}\hat{\mathscr{r}} - \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c^2\mathscr{r}} \right] d\tau'$$



$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \left[\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}^2} + \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c\mathscr{r}} \right] \times \hat{\mathscr{r}}\, d\tau'$$

$\vec{J} \leftarrow \rho\vec{v}$

$$\frac{\dot{\vec{J}}}{c^2} = \frac{\rho\dot{\vec{v}}}{c^2} = \frac{\rho}{c}\dot{\vec{\beta}}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial (ct)}$$

Point Charges

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0} \frac{\eta}{(\vec{\eta}\cdot\vec{u})^3}\left[(c^2 - v^2)\vec{u} + \vec{\eta}\times(\vec{u}\times\vec{a})\right]$$

$$\vec{u} \equiv c\hat{\eta} - \vec{v}$$

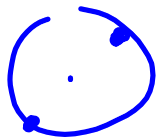
$$\vec{B}(\vec{r},t) = \frac{1}{c}\hat{\eta}\times\vec{E}(\vec{r},t)$$

Radiation

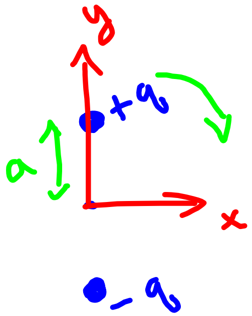
Electric / Magnetic Dipole Radiation

$$P = \frac{\mu_0 \ddot{p}^2}{6\pi c}$$ ← electric dipole moment.

$$\vec{p} = \int \vec{r}' \rho(\vec{r}')\, d\tau'$$



$T = 10\,\text{ms}$



$$\vec{p} = 2qa\,\hat{y}\cos(\omega t) + 2qa\,\hat{x}\sin(\omega t)$$

$\omega T = 2\pi$

Magnetic Dipole:

$$P = \frac{\mu_0 \ddot{m}^2}{6\pi c^2}$$

$$\vec{m} = \frac{1}{2}\int (\vec{r} \times \vec{J})\, d\tau$$

For a current loop:

$$\vec{m} = I\vec{a}$$

Point Charge Radiation

Non.rel:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

Relativistic

$$P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v}\times\vec{a}}{c}\right|^2\right)$$

Radiation Reaction

$$\vec{F}_{rad} = \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}$$