2-6.

Let the origin of our coordinate system be at the tail end of the cattle (or the closest cow/bull).

a) The bales are moving initially at the speed of the plane when dropped. Describe one of these bales by the parametric equations

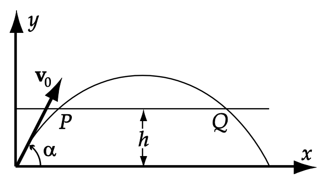
$$x = x_0 + v_0 t \tag{1}$$

$$y = y_0 - \frac{1}{2} g t^2 \tag{2}$$

where $y_0 = 80\text{ m}$, and we need to solve for x_0. From (2), the time the bale hits the ground is $\tau = \sqrt{2y_0/g}$. If we want the bale to land at $x(\tau) = -30\text{ m}$, then $x_0 = x(\tau) - v_0 \tau$. Substituting $v_0 = 44.4\text{ m}\cdot\text{s}^{-1}$ and the other values, this gives $x_0 \simeq -210\text{ m}$. The rancher should drop the bales 210 m behind the cattle.

b) She could drop the bale earlier by any amount of time and not hit the cattle. If she were late by the amount of time it takes the bale (or the plane) to travel by 30 m in the *x*-direction, then she will strike cattle. This time is given by $(30\text{ m})/v_0 \simeq 0.68\text{ s}$.

2-8.



From problem 2-3 the equations for the coordinates are

$$x = v_0 t \cos\alpha \tag{1}$$

$$y = v_0 t \sin\alpha - \frac{1}{2} g t^2 \tag{2}$$

In order to calculate the time when a projective reaches the ground, we let $y = 0$ in (2):

$$v_0 t \sin\alpha - \frac{1}{2} g t^2 = 0 \tag{3}$$

$$t = \frac{2v_0}{g} \sin\alpha \tag{4}$$

Substituting (4) into (1) we find the relation between the range and the angle as

$$x = \frac{v_0^2}{g} \sin 2\alpha \tag{5}$$

The range is maximum when $2\alpha = \frac{\pi}{2}$, i.e., $\alpha = \frac{\pi}{4}$. For this value of α the coordinates become

$$\left.\begin{aligned} x &= \frac{v_0}{\sqrt{2}} t \\ x &= \frac{v_0}{\sqrt{2}} t - \frac{1}{2} g t^2 \end{aligned}\right] \tag{6}$$

Eliminating t between these equations yields

$$x^2 - \frac{v_0^2}{g} x + \frac{v_0^2}{g} y = 0 \tag{7}$$

We can find the x-coordinate of the projectile when it is at the height h by putting $y = h$ in (7):

$$x^2 - \frac{v_0^2}{g} x + \frac{v_0^2 h}{g} = 0 \tag{8}$$

This equation has two solutions:

$$\left.\begin{aligned} x_1 &= \frac{v_0^2}{2g} - \frac{v_0^2}{2g} \sqrt{v_0^2 - 4gh} \\ x_2 &= \frac{v_0^2}{2g} + \frac{v_0^2}{2g} \sqrt{v_0^2 - 4gh} \end{aligned}\right] \tag{9}$$

where x_1 corresponds to the point P and x_2 to Q in the diagram. Therefore,

$$\boxed{d = x_2 - x_1 = \frac{v_0}{g} \sqrt{v_0^2 - 4gh}} \tag{10}$$

2-13. The equation of motion of the particle is

$$m \frac{dv}{dt} = -mk\left(v^3 + a^2 v\right) \tag{1}$$

Integrating,

$$\int \frac{dv}{v\left(v^2 + a^2\right)} = -k \int dt \tag{2}$$

and using Eq. (E.3), Appendix E, we find

$$\frac{1}{2a^2} \ln\left[\frac{v^2}{a^2 + v^2}\right] = -kt + C \tag{3}$$

Therefore, we have

$$\frac{v^2}{a^2 + v^2} = C' e^{-At} \tag{4}$$

where $A \equiv 2a^2 k$ and where C' is a new constant. We can evaluate C' by using the initial condition, $v = v_0$ at $t = 0$:

$$C' = \frac{v_0^2}{a^2 + v_0^2} \tag{5}$$

Substituting (5) into (4) and rearranging, we have

$$v = \left[\frac{a^2 C' e^{-At}}{1 - C' e^{-At}} \right]^{1/2} = \frac{dx}{dt} \tag{6}$$

Now, in order to integrate (6), we introduce $u \equiv e^{-At}$ so that $du = -Au\, dt$. Then,

$$x = \int \left[\frac{a^2 C' e^{-At}}{1 - C' e^{-At}} \right]^{1/2} dt = \frac{a}{A} \int \left[\frac{C' u}{1 - C' u} \right]^{1/2} \frac{du}{u}$$

$$= -\frac{a\sqrt{C'}}{A} \int \frac{du}{\sqrt{-C' u^2 + u}} \tag{7}$$

Using Eq. (E.8c), Appendix E, we find

$$x = \frac{a}{A} \sin^{-1}(1 - 2C'u) + C'' \tag{8}$$

Again, the constant C'' can be evaluated by setting $x = 0$ at $t = 0$; i.e., $x = 0$ at $u = 1$:

$$C'' = -\frac{a}{A} \sin^{-1}(1 - 2C') \tag{9}$$

Therefore, we have

$$x = \frac{a}{A} \left[\sin^{-1}\left(-2C' e^{-At} + 1\right) - \sin^{-1}(-2C' + 1) \right]$$

Using (4) and (5), we can write

$$x = \frac{1}{2ak} \left[\sin^{-1}\left[\frac{-v^2 + a^2}{v^2 + a^2} \right] - \sin^{-1}\left[\frac{-v_0^2 + a^2}{v_0^2 + a^2} \right] \right] \tag{10}$$

From (6) we see that $v \to 0$ as $t \to \infty$. Therefore,

$$\lim_{t \to \infty} \sin^{-1}\left[\frac{-v^2 + a^2}{v^2 + a^2} \right] = \sin^{-1}(1) = \frac{\pi}{2} \tag{11}$$

Also, for very large initial velocities,

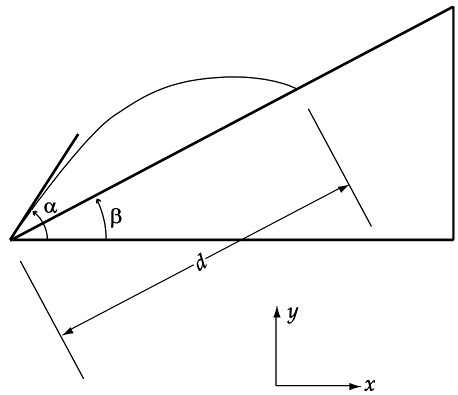
$$\lim_{v_0 \to \infty} \sin^{-1}\left[\frac{-v_0^2 + a^2}{v_0^2 + a^2} \right] = \sin^{-1}(-1) = -\frac{\pi}{2} \tag{12}$$

Therefore, using (11) and (12) in (10), we have

$$\boxed{x(t \to \infty) = \frac{\pi}{2ka}} \tag{13}$$

and the particle can never move a distance greater than $\pi/2ka$ for any initial velocity.

2-14.



a) The equations for the projectile are

$$x = v_0 \cos \alpha t$$

$$y = v_0 \sin \alpha t - \frac{1}{2} g t^2$$

Solving the first for t and substituting into the second gives

$$y = x \tan \alpha - \frac{1}{2} \frac{g x^2}{v_0^2 \cos^2 \alpha}$$

Using $x = d \cos \beta$ and $y = d \sin \beta$ gives

$$d \sin \beta = d \cos \beta \tan \alpha - \frac{g d^2 \cos^2 \beta}{2 v_0^2 \cos^2 \alpha}$$

$$0 = d \left[\frac{g d \cos^2 \beta}{2 v_0^2 \cos^2 \alpha} - \cos \beta \tan \alpha + \sin \beta \right]$$

Since the root $d = 0$ is not of interest, we have

$$d = \frac{2(\cos \beta \tan \alpha - \sin \beta) v_0^2 \cos^2 \alpha}{g \cos^2 \beta}$$

$$= \frac{2 v_0^2 \cos \alpha (\sin \alpha \cos \beta - \cos \alpha \sin \beta)}{g \cos^2 \beta}$$

$$\boxed{d = \frac{2 v_0^2 \cos \alpha \sin (\alpha - \beta)}{g \cos^2 \beta}} \qquad (1)$$

b) Maximize d with respect to α

$$\frac{d}{d\alpha}(d) = 0 = \frac{2 v_0^2}{g \cos^2 \beta} \left[-\sin \alpha \sin (\alpha - \beta) + \cos \alpha \cos (\alpha - \beta) \right] \cos (2\alpha - \beta)$$

$$\cos (2\alpha - \beta) = 0$$

$$2\alpha - \beta = \frac{\pi}{2}$$

$$\boxed{\alpha = \frac{\pi}{4} + \frac{\beta}{2}}$$

c) Substitute (2) into (1)

$$d_{\max} = \frac{2v_0^2}{g\cos^2\beta}\left[\cos\left(\frac{\pi}{4} + \frac{\beta}{2}\right)\sin\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right]$$

Using the identity

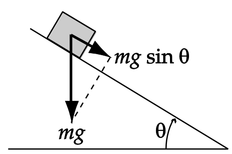
$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

we have

$$d_{\max} = \frac{2v_0^2}{g\cos^2\beta}\cdot\frac{\sin\frac{\pi}{2} - \sin\beta}{2} = \frac{v_0^2}{g}\left[\frac{1-\sin\beta}{1-\sin^2\beta}\right]$$

$$\boxed{d_{\max} = \frac{v_0^2}{g\left(1+\sin\beta\right)}}$$

2-15.



The equation of motion along the plane is

$$m\frac{dv}{dt} = mg\sin\theta - kmv^2 \tag{1}$$

Rewriting this equation in the form

$$\frac{1}{k}\frac{dv}{\frac{g}{k}\sin\theta - v^2} = dt \tag{2}$$

We know that the velocity of the particle continues to increase with time (i.e., $dv/dt > 0$), so that $\left(g/k\right)\sin\theta > v^2$. Therefore, we must use Eq. (E.5a), Appendix E, to perform the integration. We find

$$\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}\sin\theta}}\tanh^{-1}\left[\frac{v}{\sqrt{\frac{g}{k}\sin\theta}}\right] = t + C \tag{3}$$

The initial condition $v(t = 0) = 0$ implies $C = 0$. Therefore,

$$v = \sqrt{\frac{g}{k}\sin\theta}\,\tanh\left(\sqrt{gk\sin\theta}\,t\right) = \frac{dx}{dt} \tag{4}$$

We can integrate this equation to obtain the displacement x as a function of time:

$$x = \sqrt{\frac{g}{k}\sin\theta}\int \tanh\left(\sqrt{gk\sin\theta}\,t\right)dt$$

Using Eq. (E.17a), Appendix E, we obtain

$$x = \sqrt{\frac{g}{k}\sin\theta}\,\frac{\ln\cosh\left(\sqrt{gk\sin\theta}\,t\right)}{\sqrt{gk\sin\theta}} + C' \tag{5}$$

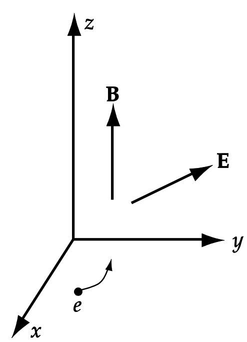
The initial condition $x(t = 0) = 0$ implies $C' = 0$. Therefore, the relation between d and t is

$$d = \frac{1}{k}\ln\cosh\left(\sqrt{gk\sin\theta}\,t\right) \tag{6}$$

From this equation, we can easily find

$$\boxed{t = \frac{\cosh^{-1}\left(e^{dk}\right)}{\sqrt{gk\sin\theta}}} \tag{7}$$

2-22.



Our force equation is

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) \tag{1}$$

a) Note that when $\mathbf{E} = 0$, the force is always perpendicular to the velocity. This is a centripetal acceleration and may be analyzed by elementary means. In this case we have also $\mathbf{v}\perp\mathbf{B}$ so that $\left|\mathbf{v}\times\mathbf{B}\right| = vB$.

$$ma_{centripetal} = \frac{mv^2}{r} = qvB \tag{2}$$

Solving this for r

$$r = \frac{mv}{qB} = \frac{v}{\omega_c} \tag{3}$$

with $\omega_c \equiv qB/m$.

b) Here we don't make any assumptions about the relative orientations of **v** and **B**, i.e. the velocity may have a component in the z direction upon entering the field region. Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, with $\mathbf{v} = \dot{\mathbf{r}}$ and $\mathbf{a} = \ddot{\mathbf{r}}$. Let us calculate first the $\mathbf{v} \times \mathbf{B}$ term.

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & B \end{vmatrix} = B\left(\dot{y}\mathbf{i} - \dot{x}\mathbf{j}\right) \tag{4}$$

The Lorentz equation (1) becomes

$$\mathbf{F} = m\ddot{\mathbf{r}} = qB\dot{y}\mathbf{i} + q\left(E_y - B\dot{x}\right)\mathbf{j} + qE_z\mathbf{k} \tag{5}$$

Rewriting this as component equations:

$$\ddot{x} = \frac{qB}{m}\dot{y} = \omega_c \dot{y} \tag{6}$$

$$\ddot{y} = -\frac{qB}{m}\dot{x} + \frac{qE_y}{m} = -\omega_c\left(\dot{x} - \frac{E_y}{B}\right) \tag{7}$$

$$\ddot{z} = \frac{qE_z}{m} \tag{8}$$

The z-component equation of motion (8) is easily integrable, with the constants of integration given by the initial conditions in the problem statement.

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m}t^2 \tag{9}$$

c) We are asked to find expressions for $\dot{x}$ and $\dot{y}$, which we will call v_x and v_y, respectively. Differentiate (6) once with respect to time, and substitute (7) for $\dot{v}_y$

$$\ddot{v}_x = \omega_c \dot{v}_y = -\omega_c^2\left(v_x - \frac{E_y}{B}\right) \tag{10}$$

or

$$\ddot{v}_x + \omega_c^2 v_x = \omega_c^2 \frac{E_y}{B} \tag{11}$$

This is an inhomogeneous differential equation that has both a homogeneous solution (the solution for the above equation with the right side set to zero) and a particular solution. The most general solution is the sum of both, which in this case is

$$v_x = C_1 \cos\left(\omega_c t\right) + C_2 \sin\left(\omega_c t\right) + \frac{E_y}{B} \tag{12}$$

where C_1 and C_2 are constants of integration. This result may be substituted into (7) to get $\dot{v}_y$

$$\dot{v}_y = -C_1\omega_c \cos\left(\omega_c t\right) - C_2\omega_c \sin\left(\omega_c t\right) \tag{13}$$

$$v_y = -C_1 \sin(\omega_c t) + C_2 \cos(\omega_c t) + K \tag{14}$$

where K is yet another constant of integration. It is found upon substitution into (6), however, that we must have $K = 0$. To compute the time averages, note that both sine and cosine have an average of zero over one of their periods $T \equiv 2\pi/\omega_c$.

$$\langle \dot{x} \rangle = \frac{E_y}{B}, \quad \langle \dot{y} \rangle = 0 \tag{15}$$

d) We get the parametric equations by simply integrating the velocity equations.

$$x = \frac{C_1}{\omega_c} \sin(\omega_c t) - \frac{C_2}{\omega_c} \cos(\omega_c t) + \frac{E_y}{B} t + D_x \tag{16}$$

$$y = \frac{C_1}{\omega_c} \cos(\omega_c t) + \frac{C_2}{\omega_c} \sin(\omega_c t) + D_y \tag{17}$$

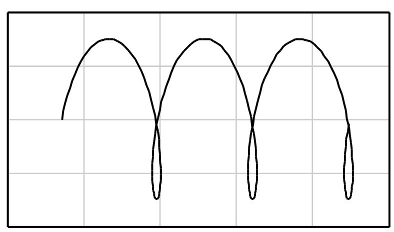
where, indeed, D_x and D_y are constants of integration. We may now evaluate all the C's and D's using our initial conditions $x(0) = -A/\omega_c$, $\dot{x}(0) = E_y/B$, $y(0) = 0$, $\dot{y}(0) = A$. This gives us $C_1 = D_x = D_y = 0$, $C_2 = A$ and gives the correct answer

$$x(t) = \frac{-A}{\omega_c} \cos(\omega_c t) + \frac{E_y}{B} t \tag{18}$$

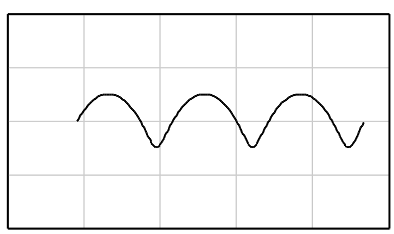
$$y(t) = \frac{A}{\omega_c} \sin(\omega_c t) \tag{19}$$

These cases are shown in the figure as (i) $A > E_y/B$, (ii) $A < E_y/B$, and (iii) $A = E_y/B$.

(i)



(ii)



(iii)