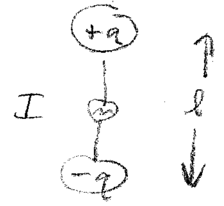
time dependent dipole: complete fields



assume $d \ll \lambda$

and $d \ll r$

but $r \approx \lambda$: looking at near field

$I = dq/dt \rightarrow p(t) = q(t)\,l$, $I(t) = \dot{p}/l$

Work from Jefimenko eqns for $\vec{E}, \vec{B}$

in terms of source ρ, J

$\vec{J}\, d^3r' \rightarrow I\, dl \rightarrow \frac{\dot{p}}{l} \cdot l = \dot{p}$

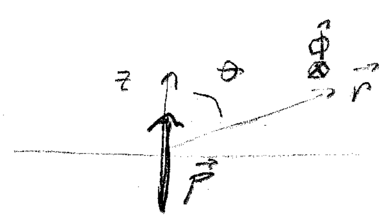
neglecting: area of wire

small length, l

magnetic field is

$$\vec{B}(\vec{r},t) = \int_V \left(\frac{[\vec{J}] \times \hat{r}}{cr^2} + \frac{[\partial_t \vec{J}] \times \hat{r}}{c^2 r} \right) d^3r'$$

$$\rightarrow \frac{[\dot{\vec{p}}] \times \hat{r}}{cr^2} + \frac{[\ddot{\vec{p}}] \times \hat{r}}{c^2 r}$$



$$= \frac{1}{cr} \left(\frac{[\dot{p}]}{r} + \frac{[\ddot{p}]}{c} \right) \sin\theta\, \hat{\phi}$$

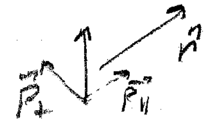
$$\vec{E}(\vec{r},t) = \int_V \left(\frac{[\rho]\hat{r}}{r^2} + \frac{[\dot{\rho}]\hat{r}}{cr} - \frac{[\partial_t \vec{J}]}{c^2 r} \right) d^3r'$$

ok on $\partial_t \vec{J}$ term, but not ρ .... $\rightarrow -\frac{[\ddot{\vec{p}}]}{c^2 r} \equiv \vec{E}_3$

must acct for t_r in ρ terms.

for arbitrary $\vec{r}$, decompose $\vec{p}$ into

a $\vec{p}_{\parallel} + \vec{p}_{\perp}$: $\vec{p}_{\parallel} \parallel \vec{r}$



$\vec{p}_{\perp} \perp \vec{r}$

on axis of $\vec{p}$

$$(\vec{E}_1)_{ax} = \frac{q(t_{r+})}{z_+^2} - \frac{q(t_{r-})}{z_-^2}$$

these are different retarded times:

let $t_r = t - r/c$ (to origin)

$t_{r\pm} = t_r \pm \Delta t$ $\qquad \Delta t = \frac{l/2}{c}$

$z_{\pm} = z \pm l/2$

Taylor expand:

upper $\quad q(t_r + \Delta t) = q(t_r) + \dot{q}(t_r)\Delta t + \dots$

↳ + b/c eval. at later retarded time.

lower $\quad q(t_r - \Delta t) = q(t_r) - \dot{q}(t_r)\Delta t + \dots$

$$(\vec{E}_1)_{ax} = \left(\frac{q + \dot{q}\, l/2c}{z - l/2} - \frac{q - \dot{q}\, l/2c}{z + l/2} \right) \hat{z}$$

add, drop 2nd order $(l^2/4)/z^2$ terms

$ql \to p$

$$\to (\vec{E}_1)_{ax} = \frac{2[\vec{p}]}{z^3} + \frac{[\dot{\vec{p}}]}{cz^2}$$

second term:

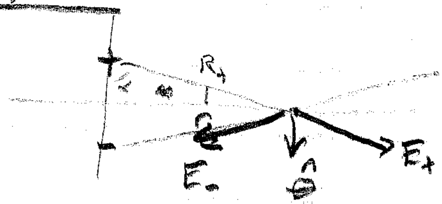
$$[\vec{E}_2]_{ax} = \left(\frac{\dot{q}(t_r+\Delta t)}{c(z-\ell/2)} - \frac{\dot{q}(t_r-\Delta t)}{c(z+\ell/2)}\right)\hat{z}$$

expand

$$\dot{q}(t_r+\Delta t) \approx \dot{q}(t_r) + \ddot{q}(t_r)\Delta t$$

$$\rightarrow [\vec{E}_2]_{ax} = \frac{[\dot{\vec{p}}]}{cz^2} + \frac{[\ddot{\vec{p}}]}{c^2 z}$$

<u>$\perp$ to $\vec{p}$ axis</u>



equidistant to charges: no retardation w/in source.

$$(\vec{E}_1)_\perp = \frac{q}{R^2}\hat{R}_+ - \frac{q}{R^2}\hat{R}_- \qquad - \frac{[\vec{p}]}{r^3}$$

$R = (r^2 + \ell^2/4)^{1/2}$

$\hat{R}_+ \cdot \hat{\theta} = \cos\alpha = \ell/2 \cdot \frac{1}{R}$

$$(\vec{E}_2)_\perp \rightarrow -\frac{[\dot{p}]}{cr^2}$$

<u>total</u>

$$\vec{E}_1 = (\vec{E}_1)_{ax}\cos\theta\,\hat{r} + (\vec{E}_1)_\perp \sin\theta\,\hat{\theta}$$

similar w/ $\vec{E}_2$

$$\rightarrow \vec{E}(\vec{r},t) = \left(\frac{2[p]}{r^3} + \frac{2[\dot{p}]}{cr^2}\right)\cos\theta\,\hat{r} + \left(\frac{[p]}{r^3} + \frac{[\dot{p}]}{cr^2} + \frac{[\ddot{p}]}{c^2 r}\right)\sin\theta\,\hat{\theta}$$

r-dependence: $1/r$ dep $\to$ far-field

$(\ddot{P}/r)\cos\theta$ terms cancelled from E_2, E_3

$1/r^3$ terms: static dipole fields

$1/r^2$ terms: mid range.

B-field: "static" term $\sim 1/r^2$

$1/r$ term is same as far field.