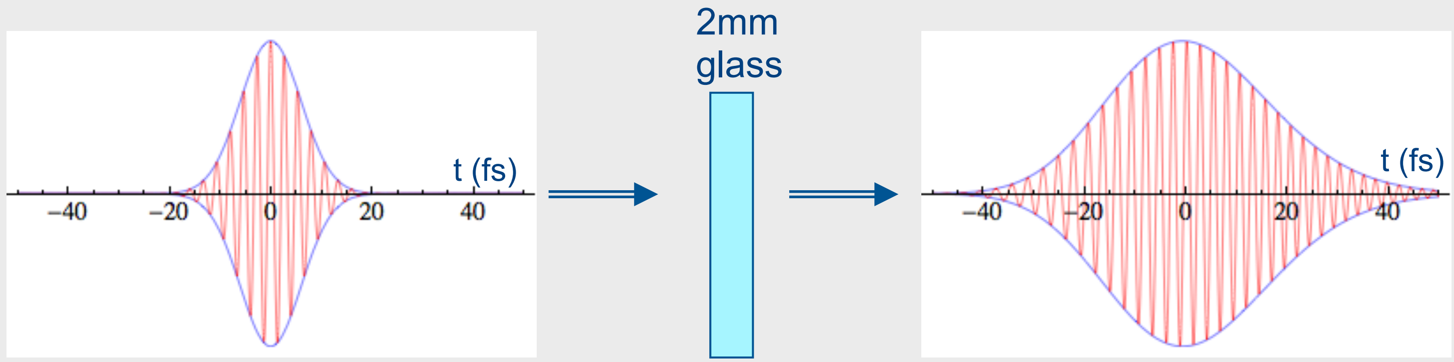
Pulse propagation: t/ω domains

- Dispersion in a system will stretch a short pulse:



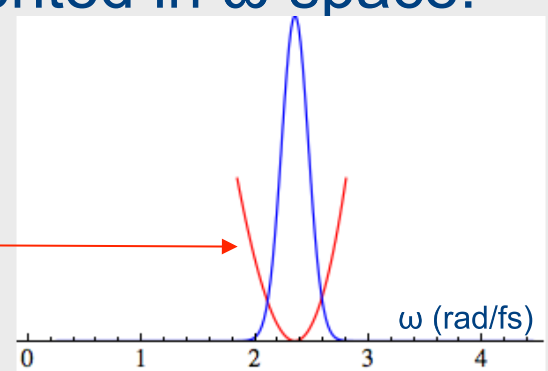
Chirped pulse

- Linear propagation is best represented in ω space:

$$E_{out}(\omega) = A(\omega - \omega_0)\, e^{i\phi(\omega)}$$

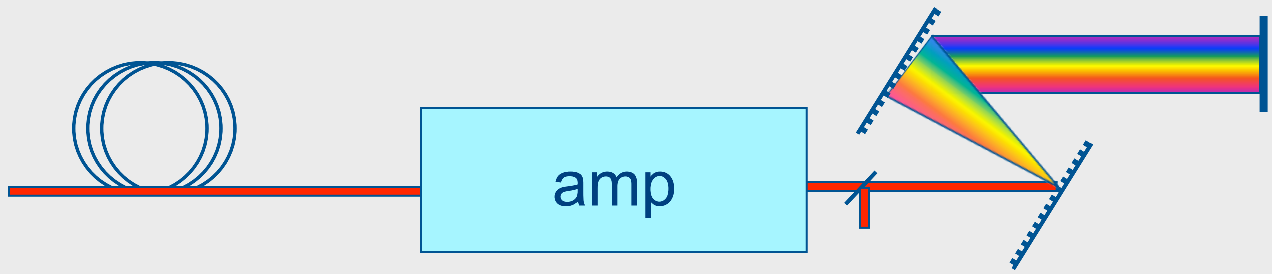
Spectral phase

$$\phi(\omega) = k\,L = \frac{\omega}{c} n(\omega)\, L$$



Stretching/compression for chirped-pulse amplification

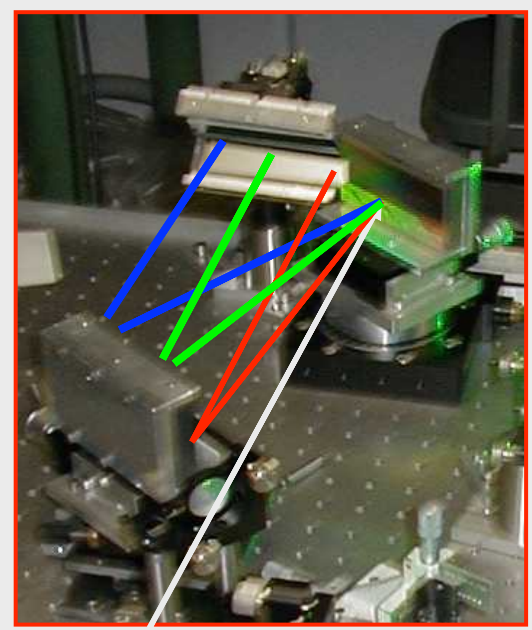
- Increase pulse duration to lower peak power
- Recompress after amplifier to get short pulse



Grating compressor: optical path depends on wavelength

spatial chirp mid-way through

temporal chirp at end



Compressor optical layout

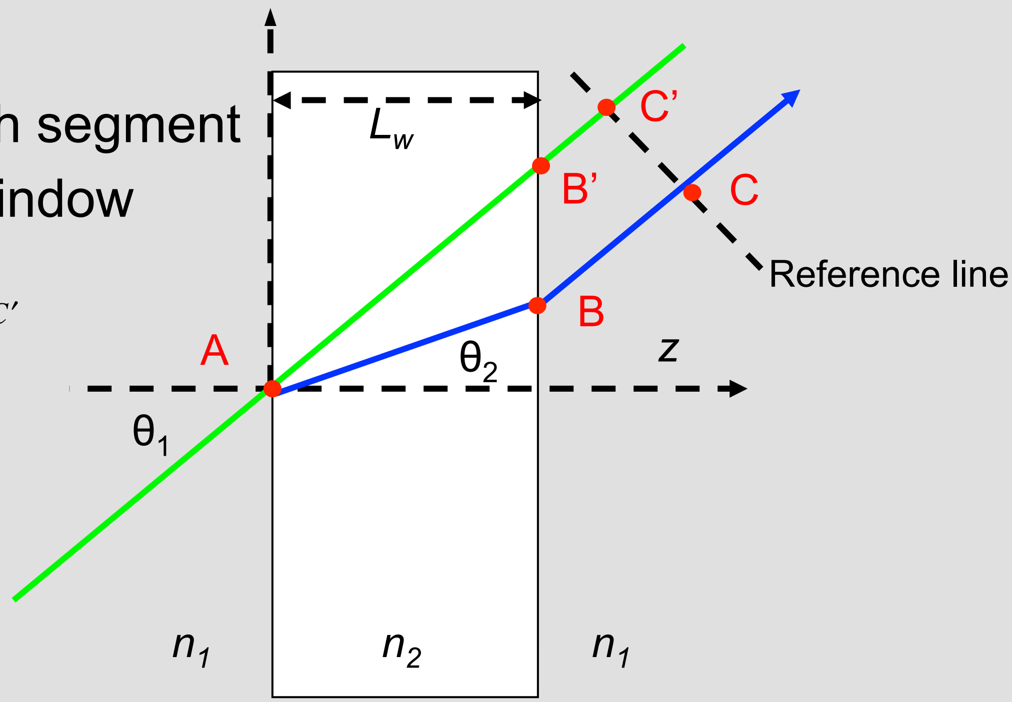
Tilted window: ray propagation

- Calculate phase shift caused by the insertion of the window into an interferometer.
- Ray optics:
  - Add up optical path for each segment
  - Subtract optical path w/o window

$$\Delta d = n_2 L_{AB} + n_1 L_{BC} - n_1 L_{AB'} - n_1 L_{B'C'}$$

$$L_{AB} = \frac{L_w}{\cos\theta_2} \qquad L_{AB'} = \frac{L_w}{\cos\theta_1}$$

$$L_{BC} = L_{B'C'} + L_{BB'} \sin\theta_1$$



  - Use Snell's Law to reduce to:

$$\phi(\omega) = \frac{\omega}{c}\Delta d(\omega) = \frac{\omega}{c} L_w \left(n_2(\omega)\cos\theta_2(\omega) - n_1(\omega)\cos\theta_1(\omega) \right)$$

Tilted window: wave propagation

- Write expression for tilted plane wave

$$E(x,z) = E_0 \exp\left[i\left(k_x x + k_z z\right)\right] = E_0 \exp\left[i\frac{\omega}{c} n\left(x\sin\theta_2 + z\cos\theta_2\right)\right]$$

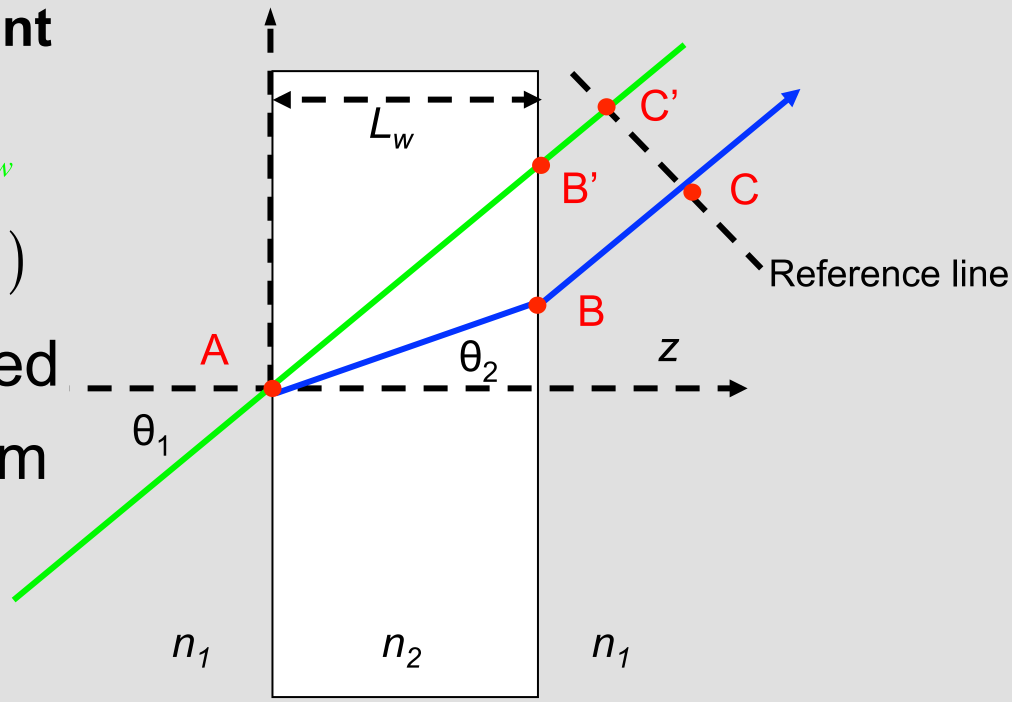
- Snell's Law: phase across surfaces is conserved

$$k_x x = \frac{\omega}{c} n \sin\theta$$ **is constant**

$$\Delta\phi = \left(k_2\cos\theta_2\right)L_w - \left(k_1\cos\theta_1\right)L_w$$

$$\Delta\phi(\omega) = \frac{\omega}{c} L_2\left(n_2\cos\theta_2 - n_1\cos\theta_1\right)$$

- This approach can be used to calculate phase of prism pairs and grating pairs

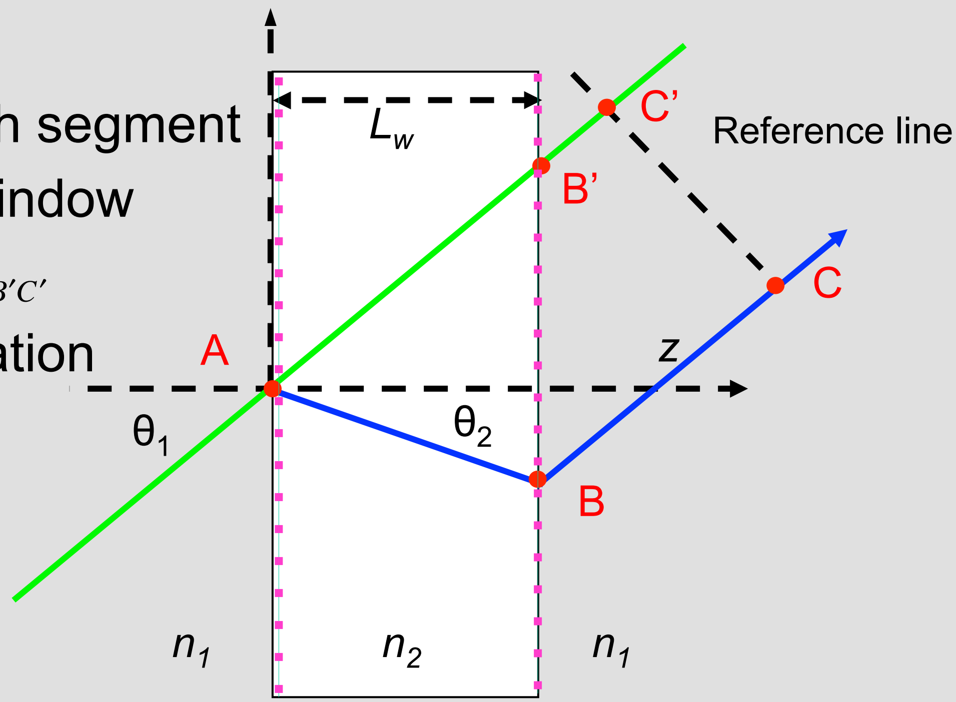


Grating pair: ray propagation

- Try to calculate phase shift by computing optical path of rays through system.
- Ray optics:
  - Add up optical path for each segment
  - Subtract optical path w/o window

$$\Delta d = n_2 L_{AB} + n_1 L_{BC} - n_1 L_{AB'} - n_1 L_{B'C'}$$

  - Use diffraction grating equation



$$n_2 \sin\theta_2 = m\frac{\lambda}{d} + n_1 \sin\theta_1$$

$$\frac{\omega}{c} n_2 \sin\theta_2 = m\frac{2\pi}{d} + \frac{\omega}{c} n_1 \sin\theta_1$$

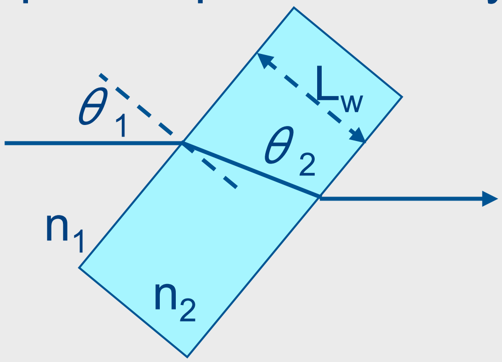
$$\phi(\omega) = \frac{\omega}{c} L_w \left(n_2 \cos\theta_2 - n_1 \cos\theta_1\right) + m\frac{2\pi}{d}\tan\theta_2$$

Extra term: this method doesn't work for gratings!

For raytracing: calculate paths, divide by local group delay.

A simple approach to calculating dispersive phase

- Two approaches:
  - Raytrace, calculate group delay
  - Calculate phase shift
- Tilted window: can be used as a basic module for constructing the spectral phase of a system

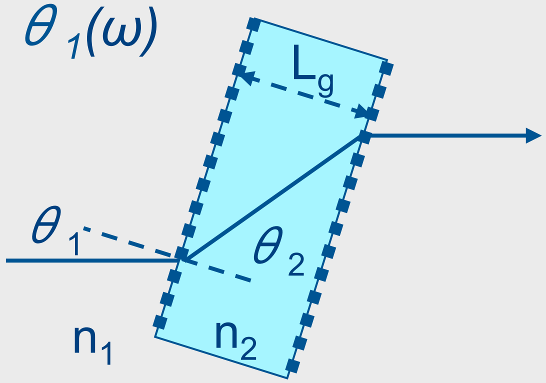


$$\phi_{window}(\lambda) = \frac{2\pi}{\lambda} L_w n_2 \cos\theta_2 - \frac{2\pi}{\lambda} L_w n_1 \cos\theta_1$$

Phase from window — Removed phase from air

- To insert inside a dispersive structure: include $\theta_1(\omega)$
- Grating compressor:

  same formalism, even with diffraction

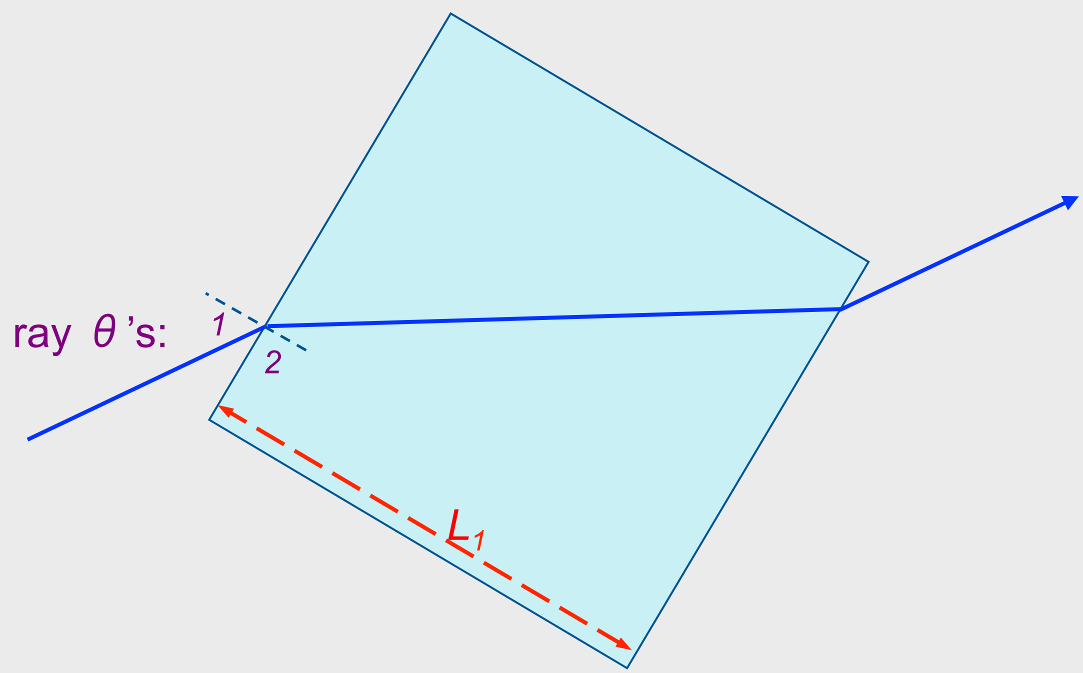


Durfee et al, Optics Express v16, 18004 (2008)

Superposition of modules: prism phase

- *Start with a tilted window of glass*

$$\phi(\omega) = \frac{\omega}{c} L_1 \left(n(\omega) \cos\theta_2(\omega) - \cos\theta_1 \right)$$

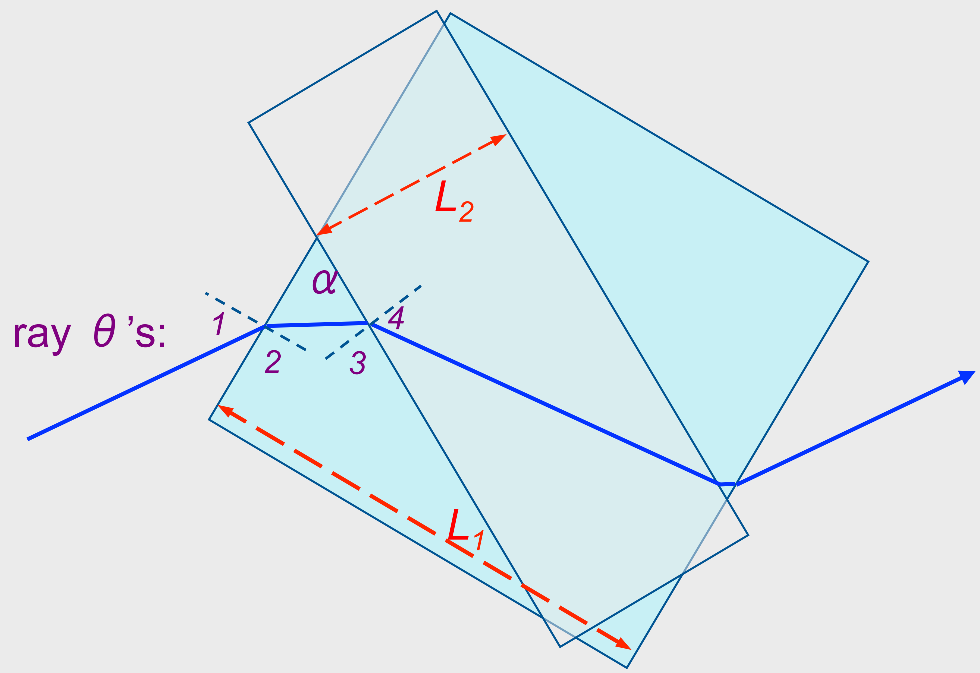


Durfee et al, Optics Express v16, 18004 (2008)

Superposition of modules: prism phase

- *Insert a "window" of air inside the glass*

$$\phi(\omega) = \frac{\omega}{c} L_1 \left(n(\omega)\cos\theta_2(\omega) - \cos\theta_1 \right) + \frac{\omega}{c} L_2 \left(\cos\theta_4(\omega) - n(\omega)\cos\theta_3(\omega) \right)$$



Durfee et al, Optics Express v16, 18004 (2008)

Superposition of modules: prism phase

- *Express phase in terms of L_p and β*

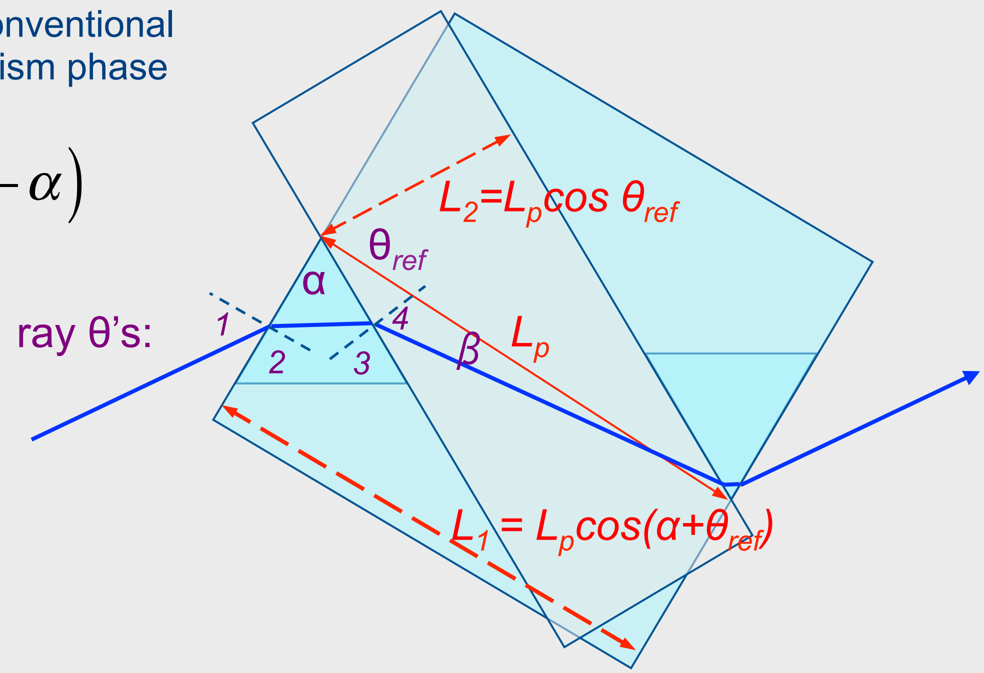
$$\phi(\omega) = \frac{\omega}{c} L_1 \left(n(\omega)\cos\theta_2(\omega) - \cos\theta_1 \right) + \frac{\omega}{c} L_2 \left(\cos\theta_4(\omega) - n(\omega)\cos\theta_3(\omega) \right)$$

$$\phi_{prisms}(\omega) = \frac{\omega}{c} L_p \cos\beta(\omega)$$

conventional prism phase

$$-\frac{\omega}{c} L_p \cos\left(\theta_1 + \theta_{ref} - \alpha\right)$$

Non-dispersive

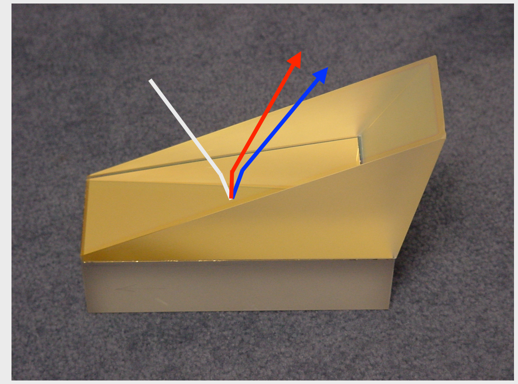


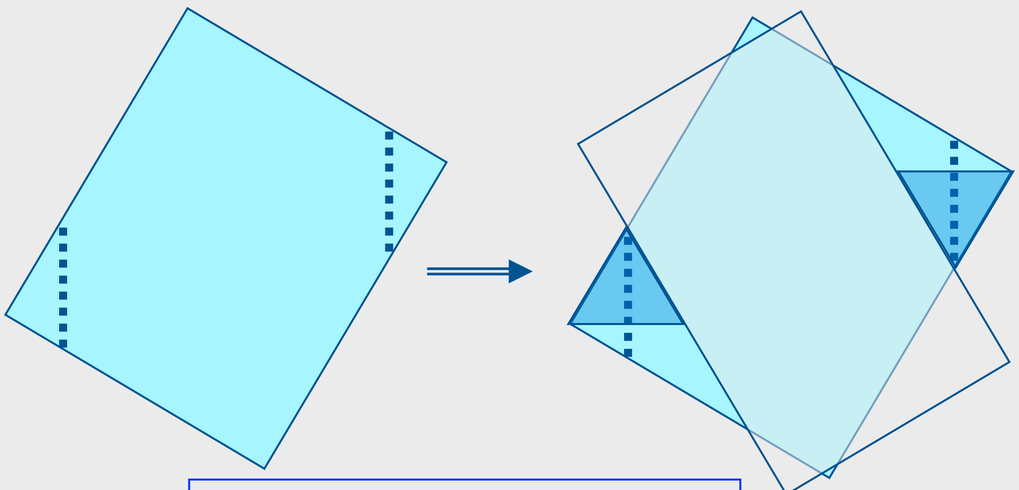
Expression reduces correctly to ϕ(tilted window) for L_2 = 0

Durfee et al, Optics Express v16, 18004 (2008)

Spectral phase of grisms

- Unfold reflection gratings to represent as a transmission gratings
- Construct grism phase as a superposition of tilted, dispersive windows





ϕ(tilted window)
+ ϕ(grating pair in glass)

+ ϕ(tilted airspace)
- ϕ(tilted glass-space)

= ϕ(prism) !

$$\phi_{grisms}(\omega) = \frac{\omega}{c} L_g \cos\beta(\omega)$$

Durfee et al, Optics Express v16, 18004 (2008)

Same form as for prism pair
Calculate exit angle using Snell and grating eqn for internal angles