

Diffractive wave propagation

Fresnel propagation

Fraunhofer propagation

Examples of diffraction

3D wave propagation

$$\nabla^2 \mathbf{E} - \frac{n_j^2}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = \frac{\partial^2}{\partial z^2} \mathbf{E} + \nabla_{\perp}^2 \mathbf{E} - \frac{n(\mathbf{r})^2}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = 0$$

- Note:

$$\nabla_{\perp}^2 = \partial_x^2 + \partial_y^2$$

$$\nabla_{\perp}^2 = \frac{1}{r} \partial_r (r \partial_r) + \frac{1}{r^2} \partial_{\phi}^2$$

- All linear propagation effects are included in LHS: diffraction, interference, focusing...
 - Previously, we assumed plane waves where transverse derivatives are zero.
- More general examples:
 - Gaussian beams (including high-order)
 - Waveguides
 - Arbitrary propagation
 - Can determine discrete solutions to linear equation (e.g. Gaussian modes, waveguide modes), then express fields in terms of those solutions.

Diffractive propagation

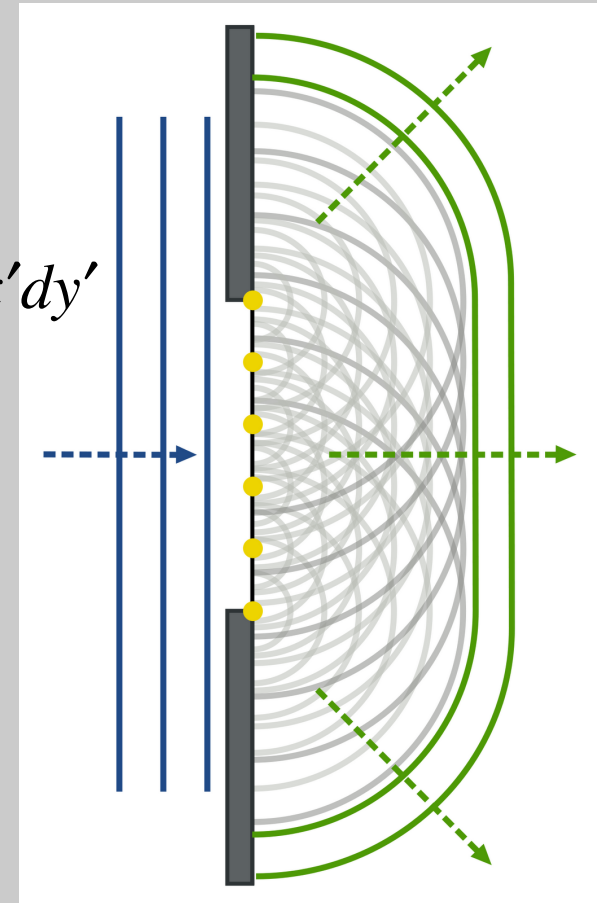
- Huygens' principle:
 - Represent a plane wave as a superposition of source points emitting spherical waves
- Integral representation:

$$E(x, y, z) = \frac{i}{\lambda} \iint E(x', y', z') \frac{\exp[-ik|\mathbf{r} - \mathbf{r}'|]}{|\mathbf{r} - \mathbf{r}'|} \cos\theta dx' dy'$$

Field at input plane Spherical wavelet Inclination factor

(Note: A blue arrow points from the label 'Inclination factor' to the $\cos\theta$ term in the equation.)

This is essentially a convolution of the complex input field with the spherical wavelets, which are the Green's function for the wave equation



Paraxial, slowly-varying approximations

- Assume

- waves are forward-propagating:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}) e^{i(kz - \omega_0 t)} + \text{c.c.}$$

- Refractive index is isotropic

$$\frac{\partial^2}{\partial z^2} \mathbf{A} + 2ik \frac{\partial}{\partial z} \mathbf{A} - k^2 \mathbf{A} + \nabla_{\perp}^2 \mathbf{A} + \frac{n^2 \omega_0^2}{c^2} \mathbf{A} = 0$$

- Fast oscillating carrier terms cancel (blue)

- Slowly-varying envelope: compare red terms

- Drop 2nd order deriv if $\frac{2\pi}{\lambda} \frac{1}{L} A \gg \frac{1}{L^2} A$

- This ignores:

- Changes in z as fast as the wavelength
- Counterpropagating waves

$$2ik \frac{\partial}{\partial z} \mathbf{A} + \nabla_{\perp}^2 \mathbf{A} = 0$$

Fresnel diffraction integral

- Fresnel approximation (near field)
 - Expand the spherical wave in paraxial approximation (in exponential)
 - Let denominator be $|\mathbf{r} - \mathbf{r}'| \sim z - z' = L \quad \cos\theta \simeq 1$
 - Input field: $E(x', y', z') = u(x', y', z') e^{+ik(z-z')}$

$$u(x, y, z) = \frac{i}{\lambda L} \iint u(x', y', z') \exp\left[-ik \frac{(x-x')^2 + (y-y')^2}{2L}\right] dx' dy'$$

$$u(x, y, z) = \frac{i}{\lambda L} e^{-ik \frac{x^2+y^2}{2L}} \iint u(x', y', z') e^{-ik \frac{x'^2+y'^2}{2L}} e^{+i \frac{k}{L}(xx'+yy')} dx' dy'$$

Fraunhofer diffraction integral

$$u(x, y, z) = \frac{i}{\lambda L} e^{-ik \frac{x^2 + y^2}{2L}} \iint u(x', y', z') e^{-ik \frac{x'^2 + y'^2}{2L}} e^{+i \frac{k}{L}(xx' + yy')} dx' dy'$$

- In the “far field”, we approximate the sum of paraxial spherical waves as a sum of plane waves
 - Assume field in input plane is confined to a radius a
 - If $\frac{ka^2}{2L} = \frac{\pi a^2}{\lambda} \frac{1}{L} \ll 1$ then we drop quadratic phases.

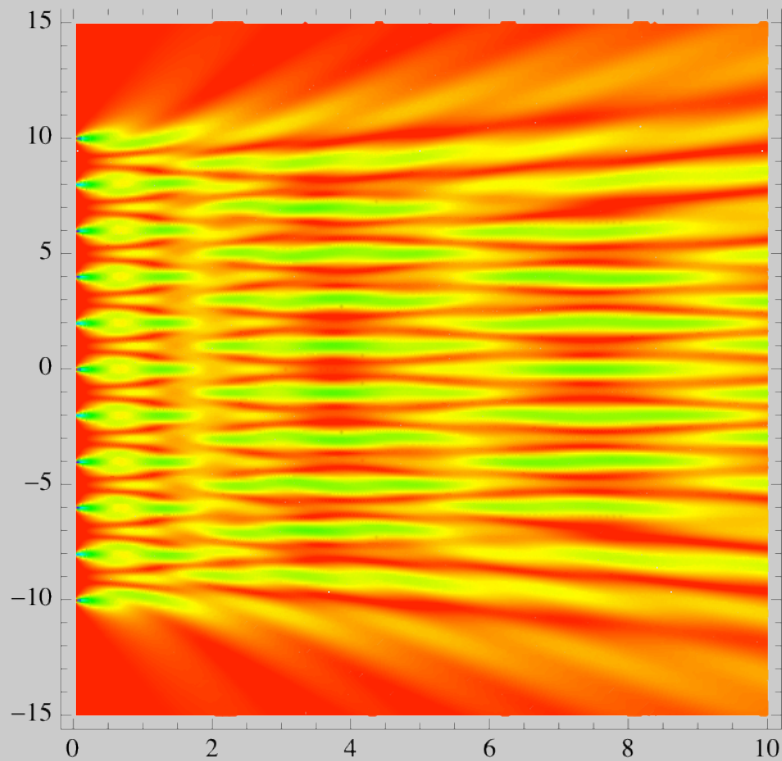
$$u(x, y, z) = \frac{i}{\lambda L} \iint u(x', y', z') \exp \left[+i \left(\frac{kx}{L} x' + \frac{ky}{L} y' \right) \right] dx' dy'$$

- Result: far field is a Fourier transform of the input field
- “spatial frequencies” $\beta_x = k \frac{x}{L} = k \sin \theta_x$ $\beta_y = k \frac{y}{L} = k \sin \theta_y$

Example: sum of dipole radiators

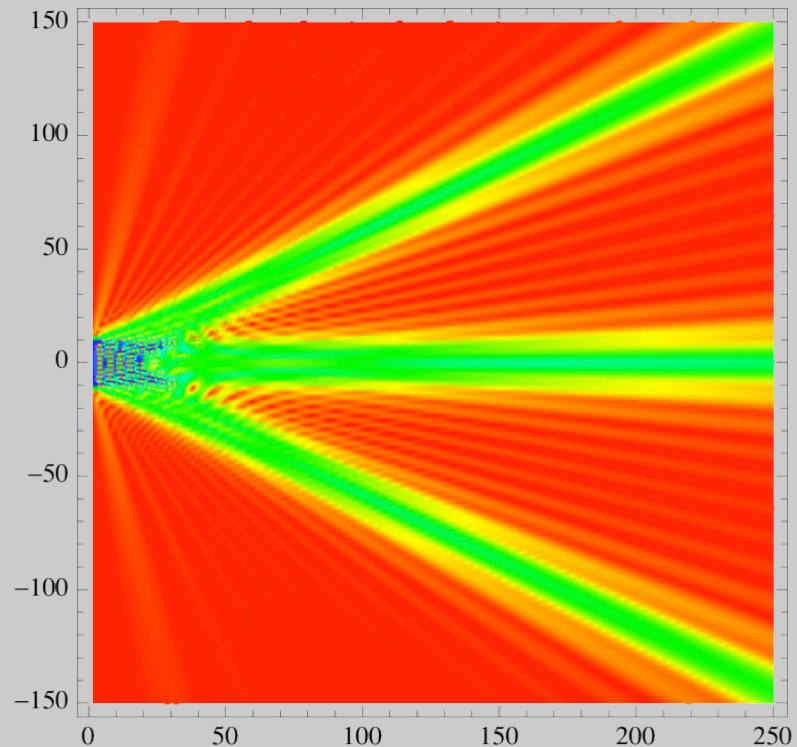
- Add fields from 10 individual sources

Near field



Talbot fringes

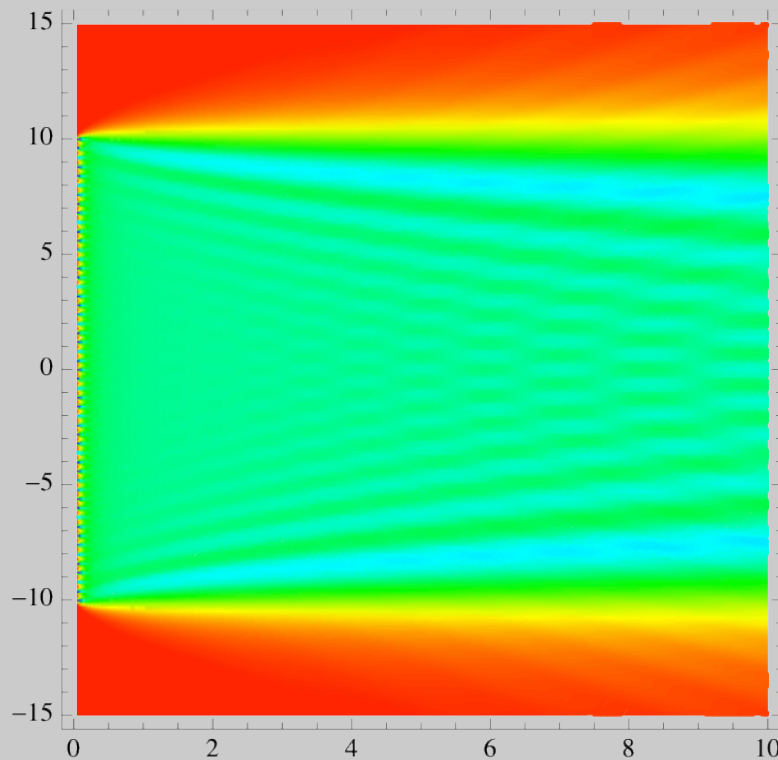
far field



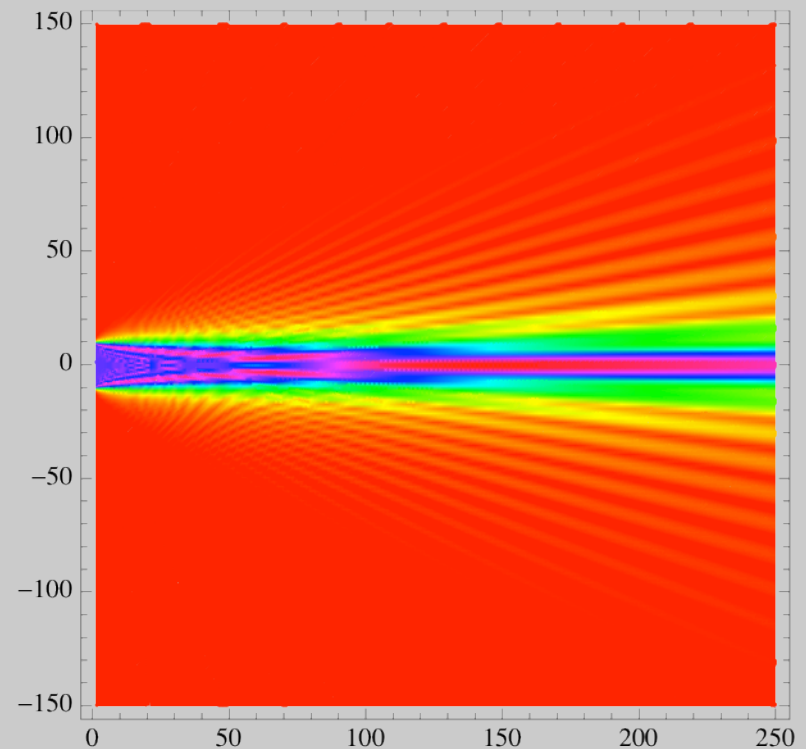
Diffraction grating

High-density of radiators

- Combine 50 sources over same distance



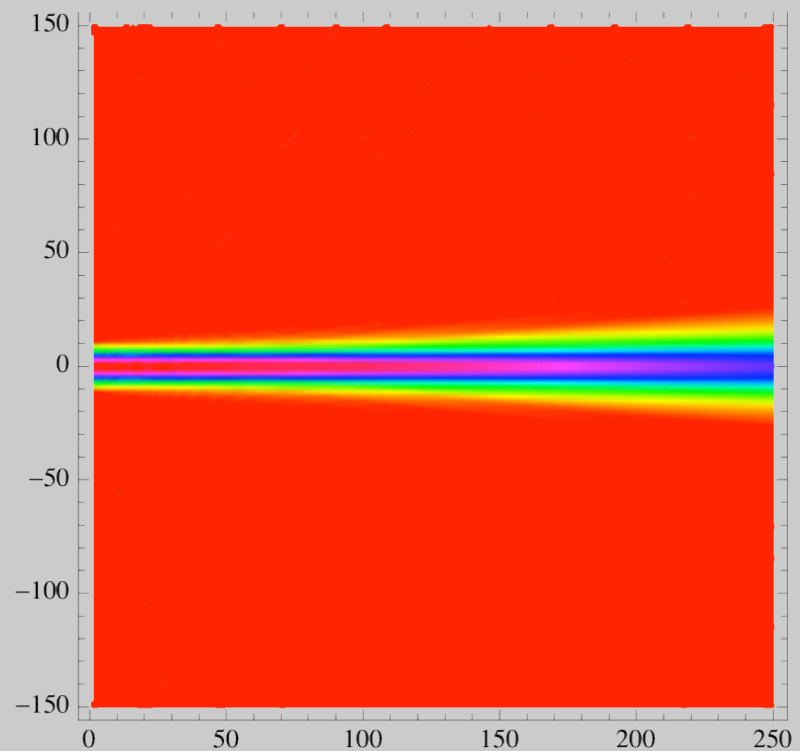
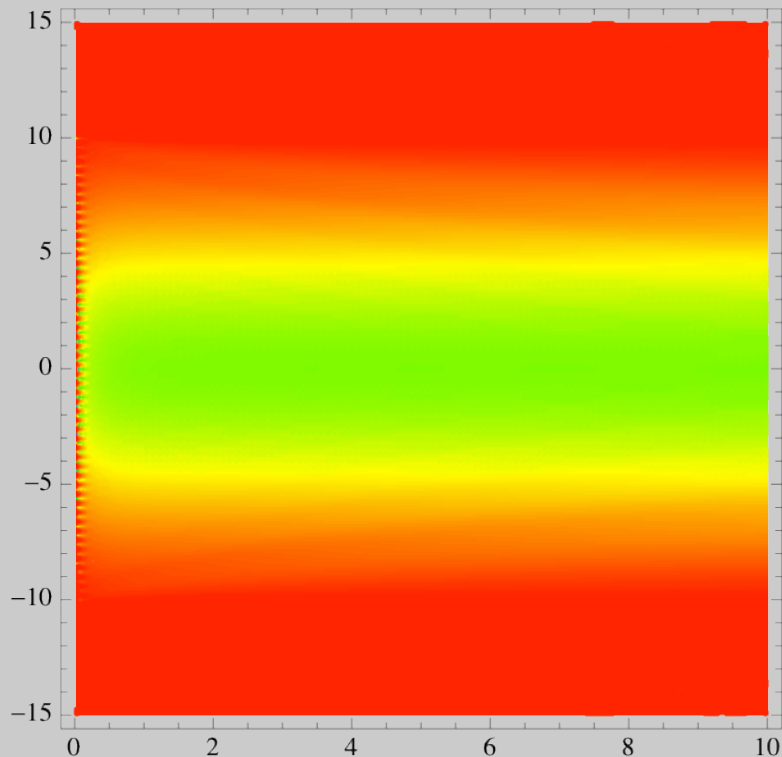
Fresnel zone shows shadow boundary, diffraction fringes



Far field evolves more like a beam, with single-slit diffraction.

High density of radiators, Gaussian envelope

- Gaussian amplitude envelope eliminates diffraction fringes



Beam smoothly spreads
out with distance