Phys 361 Homework 10

1) We're starting to get into some territory where curls are not always zero, so check this out:

a) What if you had a potential function that was only a function of φ, $V(\varphi)$? When you stop and think about, that sounds like it may be worrisome because the resulting E-field will only have a component in the $\hat{\varphi}$ direction, which seems awfully curly. Write down what that E-field will look like in terms of derivatives of V. Then show via explicit calculation that $\nabla \times \vec{E} = 0$, despite how it may look. Work in cylindrical coordinates for convenience.

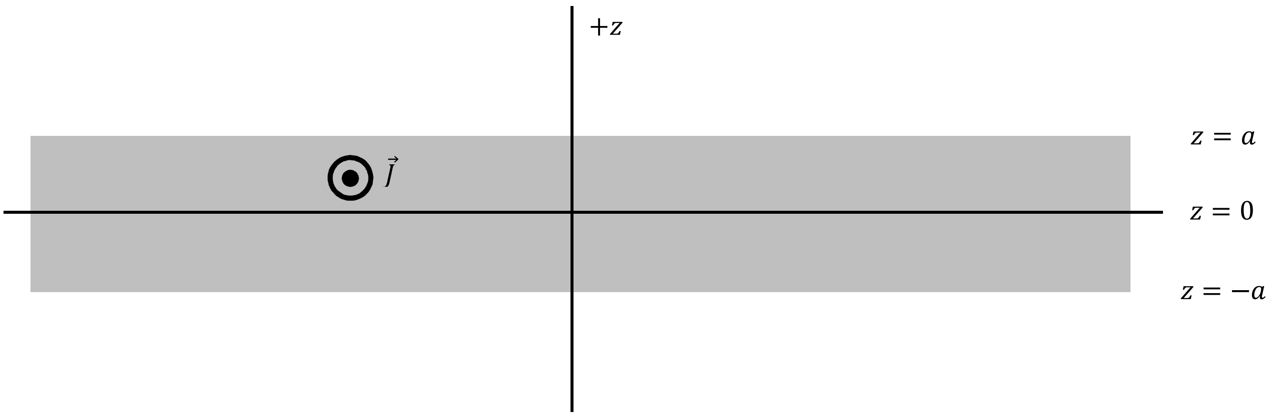
b) It turns out it's not possible in electrostatics to write a potential function that leads to an electric field with curl. Why not? (I'm just looking for a short re-statement of old ideas, so don't overdo this part)

c) Okay, so let's pick a particular potential function, $V(\varphi) = A\varphi$. Calculate and sketch the vector field that results from this potential. We know that field doesn't have curl (calculate $\vec{\nabla} \times \vec{E}$ if you don't believe me). And if $\vec{\nabla} \times \vec{E} = 0$, we know $\oint \vec{E} \cdot \vec{dl}$ is supposed to be equal to zero for any closed path. Explicitly calculate $\oint \vec{E} \cdot \vec{dl}$ for a circular closed loop of some arbitrary radius R.

d) If you did part c) right, you should be kind of worried right know. Resolve this apparent contradiction. You should be able to make some progress by examining Stokes' theorem closely in this context. Alternately (or complementarily), dredge up what you (might have) learned about contour integration in math phys and adapt it to this situation (I say adapt because you first saw it in the complex plane, but it works about the same in the real plane – you don't have to prove this). This is a deep issue, and I'd like to see some quality discussion here.

2) (based on Pollack and Stump 8.17)

We have a slab-shape object of thickness $2a$ in z carrying some current density **J** out of the page as shown:



The current density is nonuniform, and is given by $\vec{J}(z) = \frac{j_0|z|}{a}\hat{\imath}.$

Find the magnetic field inside and outside of the slab. Make sure you explain any necessary symmetry arguments in detail, since this is the first homework problem to involve Ampere's law. Do not wave the phrase "by symmetry" around like a magic wand.

3) We've had a couple discussions now on how 2-D and 3-D sources relate (like σ vs. ρ or **J** vs. **K**. My contention is that quantities of different dimensionality are independent; you can't say that a σ is just a ρ evaluated at an edge, or something. But you can *transition* from a 3-D object to a 2-D object in a limit.

a) First, consider a very wide slab of insulating material with thickness h in the z direction and uniform volume charge density ρ. Find the electric field the slab makes as a function of z (both inside and outside the slab; let $z = 0$ be the center of the slab and all other objects in this problem). Then consider a 2-D sheet of insulating material with surface charge density σ such that the sheet has the same amount of charge per area (area in the x-y plane) as the slab. The sheet (an is also at $z = 0$. Find the electric field it makes in terms of ρ (not σ) and h. Graph the electric fields for both objects as functions of z, and put said graphs side by side for easy comparison.

b) Now consider a wide slab of current coming out of the page, with some thickness h and uniform volume current density $\mathbf{J}$. Figure out the magnetic field as a function of z. Next, consider a 2-D sheet of current coming out of the page, with surface current $\mathbf{K}$ such that the sheet has the same amount of current per area as the slab. Figure out what kind of magnetic field it makes in terms of $\mathbf{J}$ and h. Graph the magnetic fields for both objects as functions of z, and put said graphs side by side.

c) Look at what happens in the limit as the slabs in a) and b) become thin, and comment on the transition from 2-D to 3-D. Among other things, make sure the general boundary conditions we have involving $\mathbf{E}$ and $\mathbf{B}$ work out.