Nonlinear Optics
Homework 1
due Monday, 24 Jan 2011

- **Problem 1: Boyd 1.4**
  - The first part comes directly from evaluating the Kroneker deltas in the equation.
  - For the explanation, consider how the nonlinear restoring force changes when driven in different directions.

- **Problem 2: Boyd 1.5**
  - to estimate the order of magnitude of the contributions, use the scaling arguments discussed in the reading. Assume there are no resonances for this part.

- **Problem 3: Boyd 1.12**
  This problem is similar to an induced grating experiment (see intro slides).
  Make a sketch of the directions of the beams that are driven by the nonlinear polarization

- **Problem 4:**
  The commonly-used nonlinear crystal BBO (beta- barium borate) is a uniaxial crystal with nonlinear coefficients d31 = 0.16 pm/V and d22 = 2.2 pm/V. The effective nonlinear coefficients as a function of the direction of the fundamental beam for this crystal are given in equation 1.5.30. The angle θ is fixed according to phase matching, but we are free to choose whatever value of ϕ that will optimize the magnitude of the nonlinearity $|d_{eff}|$.

  For an input wavelength of 800nm, the type I phase matching angle is θ = 29 deg and for type II, the angle is θ = 42.3 deg.
  - Plot d_{eff} vs ϕ for both cases, and determine the optimum value.
  - Which phase matching type gives a higher nonlinearity?

- **Problem 5:**
  We can numerically solve differential equations without a perturbation approximation. Use NDSolve[ ] in *Mathematica* (or another differential equation solver if you're using another program) to solve for the displacement of the classical nonlinear oscillator (equation 1.4.1). In *Mathematica*, use the help on NDSolve for examples of how to program it. It is actually quite easy:

```
solution = NDSolve[{x''[t] + 2 γ x'[t] + ω0^2 x[t] + a x[t]^2 == - Cos[ω t],
    x[0] == 0, x'[0] == 0}, x, {t, 0, tMax}];
xSolution[t_] = x[t] /. solution;
```

  Use ω0 = 3; ω = 1; γ = 0.1;
  a. Make plot of x(t) for a = 0, a=0.03 and a = 0.3. Explain the origin of the transients you see at early times.

  b. The solutions for a > 0 are not that different for the linear case. Subtract the linear solution function from the nonlinear solution for the two cases, and plot the two results in a region of time after the transients have died down (say after t>50). Discuss the difference between the small and large nonlinearity cases.