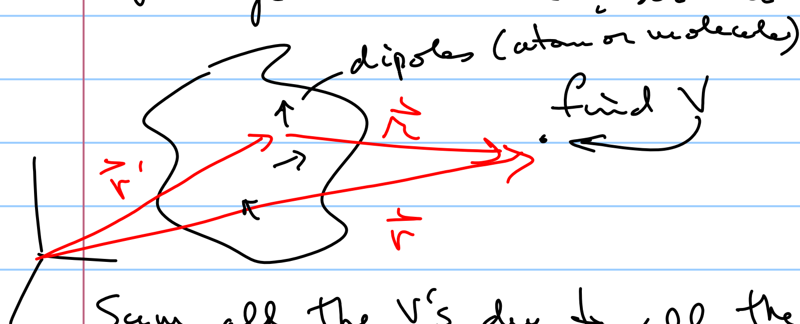
Lecture 17

Note Title 2/24/2006

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad (dq = \rho\, d\tau) \longrightarrow \frac{1}{4\pi\epsilon_0}\underbrace{\int \frac{\rho\, d\tau}{r}}_{\text{volume}} + \frac{1}{4\pi\epsilon_0}\underbrace{\int \frac{\sigma\, da}{r}}_{\text{surface}}$$

voltage from volume & surface charges



$$\underbrace{V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}}_{\text{coordinate indep.}}$$

Sum all the V's due to all the dipoles each of which has $V \propto \frac{\vec{p}\cdot r}{r^3}$

$d\vec{p} = \vec{P}\, d\tau$ ← dipole moment per volume

$$V = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \longrightarrow dV = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}\cdot(\vec{r}-\vec{r}')}{|r-r'|^3}\, d\tau'$$

$$\vec{\nabla}\Bigg(\underbrace{\frac{1}{\vec{r}-\vec{r}'}}_{\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}}\Bigg) = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} = -\frac{\vec{r}}{r^3}$$

$$\vec\nabla'\left(\frac{1}{|\vec r-\vec r'|}\right) = \frac{\vec{\mathscr{r}}}{\mathscr{r}^3} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\vec P\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau' = \frac{1}{4\pi\epsilon_0}\int \underbrace{\vec P\cdot\vec\nabla'\frac{1}{|\vec r-\vec r'|}}_{\left(\vec\nabla\cdot\frac{\vec P}{|\vec r-\vec r'|} - \frac{1}{|\vec r-\vec r'|}\vec\nabla\cdot\vec P\right)}\,d\tau'$$

$$\vec\nabla\cdot(f\vec A) = f\,\vec\nabla\cdot\vec A + \vec A\cdot\vec\nabla f$$

$$\vec A \leftrightarrow \vec P \qquad f \leftrightarrow \frac{1}{|\vec r-\vec r'|}$$

$$V = \frac{1}{4\pi\epsilon_0}\underbrace{\int \vec\nabla'\cdot\left(\frac{\vec P}{|\vec r-\vec r'|}\right)d\tau'}_{\frac{1}{4\pi\epsilon_0}\oint \frac{\vec P\cdot d\vec a}{|\vec r-\vec r'|}} - \frac{1}{4\pi\epsilon_0}\int \frac{\vec\nabla'\cdot\vec P}{|\vec r-\vec r'|}\,d\tau'$$

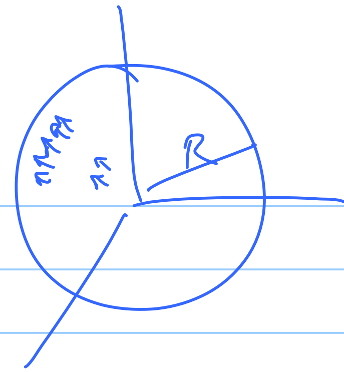
$$\frac{1}{4\pi\epsilon_0}\oint \frac{\vec P\cdot d\vec a}{|\vec r-\vec r'|} + \frac{1}{4\pi\epsilon_0}\int \frac{-\vec\nabla'\cdot\vec P}{|\vec r-\vec r'|}\,d\tau'$$

$$\sigma_b = \vec P\cdot\hat n \qquad\qquad \rho_b = -\vec\nabla'\cdot\vec P$$

$$d\vec a = \hat n\,|d\vec a|$$

$$\vec\nabla\cdot\vec A = \frac{1}{r^2}\frac{\partial}{\partial r}r^2 A_r + \cdots$$

Ex: $\vec{P} = k\vec{r}$



$$\sigma_b = \vec{P}\cdot\hat{n}\Big|_{r=R} = k\vec{r}\cdot\hat{r}\Big|_{r=R}$$

$\hat{n} \to \hat{r}$

$$\sigma_b = kR$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -3k \quad \text{constant}$$

Gauss's law gives fields

Total charge $\sigma_b 4\pi R^2 + \rho_b \frac{4}{3}\pi R^3$ should be zero