Reading today: G 9.1

Tomorrow: G 9.2

Wave Equation:

Schrödinger: $\underbrace{-\frac{\hbar^2}{2m}\nabla^2\psi + (V-E)\psi = \emptyset}_{\text{Describes waves.}}$

Wave Equation in general:

$$\boxed{\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Show that any function $g(x-v_x t)$ satisfies the 1D wave eqn $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v_x^2}\frac{\partial^2 f}{\partial t^2}$.

$$\frac{\partial^2 g}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 g}{\partial t^2}$$

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial (x-vt)}\frac{\partial \overbrace{(x-vt)}^{u}}{\partial x} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial g}{\partial u}$$

$$\frac{\partial u}{\partial x} = 1 \quad ; \quad \frac{\partial u}{\partial t} = -v$$

$$\frac{\partial^2 g}{\partial x} = \frac{\partial^2 g}{\partial u^2}\frac{\partial u}{\partial x} = \frac{\partial^2 g}{\partial u^2}$$

$$\frac{\partial^2 g}{\partial x^2} = \frac{\partial^2 g}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2$$ only if $u(x)$ is linear in x.

It's linear in t so (u)

$$\frac{\partial^2 g}{\partial t^2} = \frac{\partial^2 g}{\partial u^2}\left(\frac{\partial u}{\partial t}\right)^2 = v^2\frac{\partial^2 g}{\partial u^2}$$

$$1 = 1$$

$(x+vt)$ left going wave

$(-x-vt)$ left

Sin functions are your friends.

Assume your wave looks like

$$f(z,t) = A\cos(kz - \omega t + \delta)$$ phase const.

$$v_{wave} = \frac{\omega}{k}$$

angular frequency $\omega = 2\pi f$

propagation const or wave number

Euler's Eqn: $e^{i\theta} = \cos\theta + i\sin\theta$

$$f(z,t) = Re\left[A e^{i(kz-\omega t+\delta)}\right]$$

$$= Re\Big[\underbrace{\underbrace{Ae^{i\delta}}_{\tilde{A}}\, e^{i(kz-\omega t)}}_{\tilde{f}}\Big]$$

$$\tilde{f}(z,t) = \tilde{A} e^{i(kz-\omega t)}$$

$$f_1 + f_2 = \; = Re[\tilde{f}_1] + Re[\tilde{f}_2]$$

$$= Re[\tilde{f}_1 + \tilde{f}_2]$$

Any function of t can be written as

$$\tilde{f}(t) = \int_{-\infty}^{\infty} \tilde{A}(\omega) e^{-i\omega t}\, d\omega$$

Any wave can be written by replacing

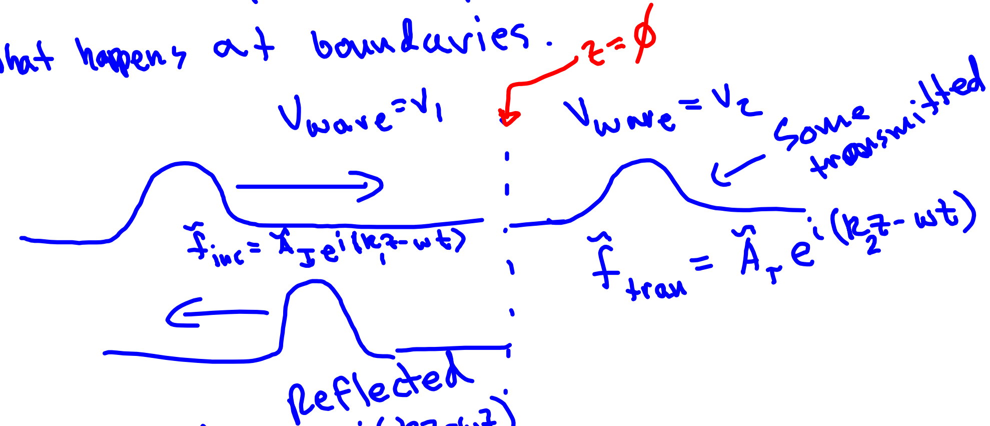
$\omega t \to kx - \omega t$

$$\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(\omega) e^{i(kz-\omega t)}\, d\omega$$

What can waves do?

Reflect, Transmit, travel

What happens at boundaries.



$$\tilde{f}_{inc} = \tilde{A}_I e^{i(k_1 z - \omega t)}$$

$$\tilde{f}_{tran} = \tilde{A}_T e^{i(k_2 z - \omega t)}$$

$$\tilde{f}_{ref} = \tilde{A}_R e^{i(-k_1 z - \omega t)}$$

Solve for $\tilde{A}_R, \tilde{A}_T$: BCs:

$$\tilde{f}(0^-, t) = \tilde{f}(0^+, t)$$

$$\frac{\partial \tilde{f}}{\partial z}(0^-, t) = \frac{\partial \tilde{f}}{\partial z}(0^+, t)$$

$$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{A}_I \; ; \quad \tilde{A}_T = \left(\frac{2v_2}{v_2 + v_1}\right)\tilde{A}_I$$