MACS 315 - May 17, 2007	**NAME**:
Exam I - 50 Points - 50 minutes	**SECTION**:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) A psychology student wants to run an experiment on color blindness. His professor tells him that when colorblind mice mate they have colorblind children. The student decides to grow his own population of colorblind mice in his parents' basement. Assume that the basement has an unlimited size, and unlimited food resources. Also assume that, over the course of the experiment, no mice leave the basement and no mice die.

   (a) Assume that the mice population, $P(t)$, grows at a rate proportional to the current population. Write down a differential equation that models the colorblind mice population.

   (b) If the student starts off with four colorblind mice and has 16 mice after two days then how many mice will the student have after seven days?

   (c) Assume now that the student has only enough resources to maintain a population of 10 mice. Change the differential equation from (a) to reflect this. Find two solutions to this differential equation. YOU SHOULDN'T SEPARATE VARIABLES.

   (d) Assume now that five mice leave the basement per unit time. Change the differential equation from (c) to model this population. Do not solve the differential equation.

2. (10 Points) Given,

(a) $t\dfrac{dy}{dt} + \cos^2(t)y = 0$ (b) $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$ (c) $\dfrac{dy}{dt} + ty^2 = 0$

For each of the previous ODE's determine its **order** and whether the ODE is **linear**, **homogenous**, and **separable**. (SEPARABILITY SHOULD ONLY APPLY TO 1^{st} ORDER ODE'S)

3. (10 Points) Verify that $y(t)$ is a solution to the associated ODE.

(a) The function, $y(t) = 3t + t^2$, for the ODE, $t\dfrac{dy}{dt} - y = t^2$.

(b) The function, $y(t) = \sin(t)$, for the ODE, $\dfrac{d^2y}{dt^2} + y = 0$.

(c) The function, $y(t) = \cos(t)$, for the ODE, $\dfrac{d^2y}{dt^2} + y = 0$.

4. (10 Points) Use separation of variables find all solutions to the following differential equations. When given an initial condition, determine the value of the constant of integration.

   (a) $\dfrac{dy}{dt} = (y^2 t + y^2)\cos(t), \quad y(0) = -1.$

   (b) $\dfrac{dy}{dt} = y(1-y)$

5. (10 Points) Given,

   (a) $\dfrac{dy}{dt} = 3y(1-y)$ (b) $\dfrac{dy}{dt} = \left(y + \dfrac{1}{2}\right)(y+t)$ (c) $\dfrac{dy}{dt} = y^2 - 1.$

   For each of the previous ODE's determine which of the following slope fields correspond to the ODE. (FOR SOME ODE'S THERE MAY NOT BE A POSSIBLE MATCH)