Resonator mode analysis

Analysis of resonators

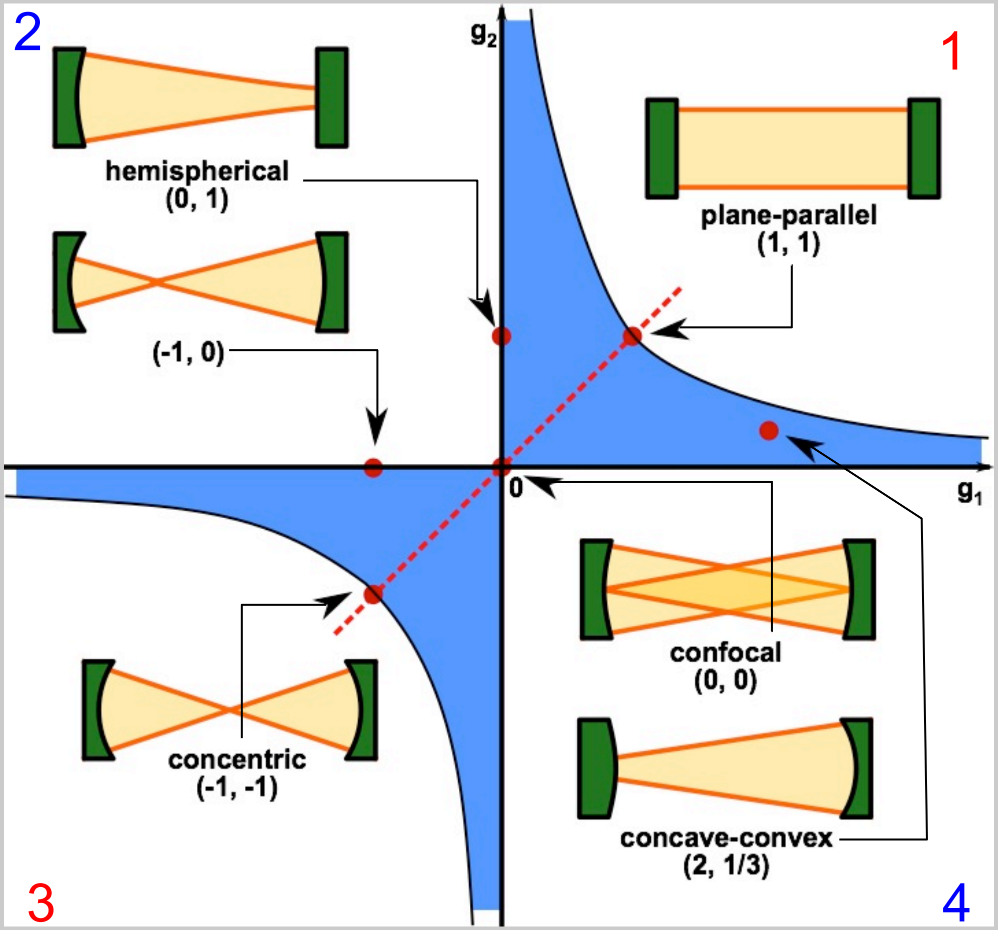
- beam sizes
- beam waist position

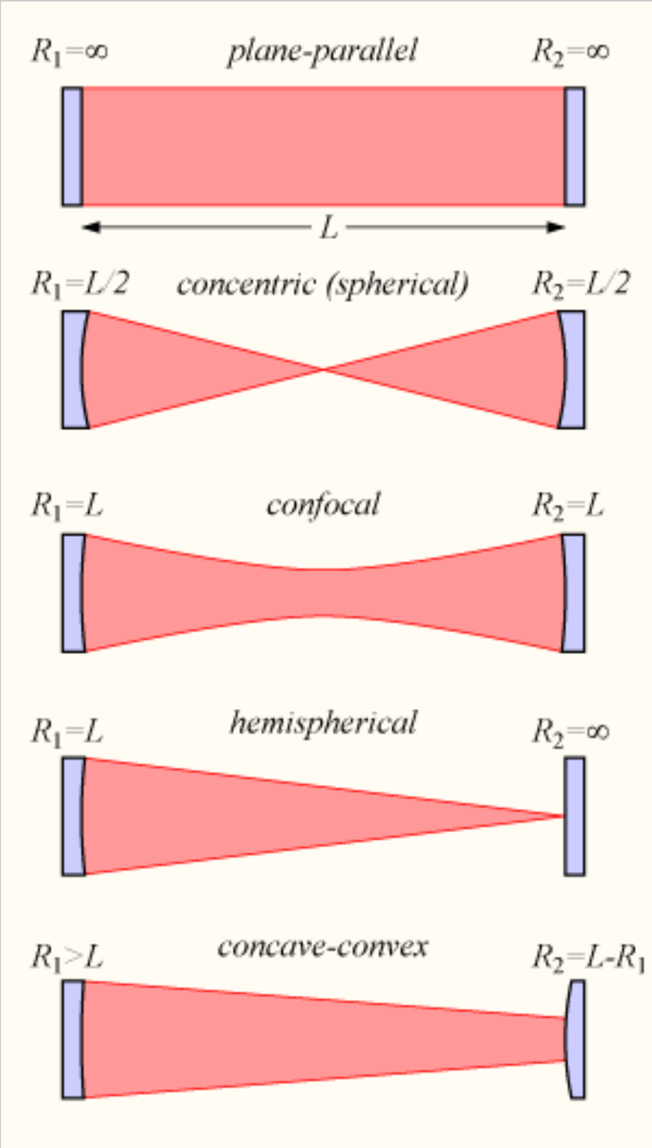
Examples of resonator design

Boundaries of stability

$$g_1 = 1 - \frac{L}{R_1} \quad g_2 = 1 - \frac{L}{R_2}$$

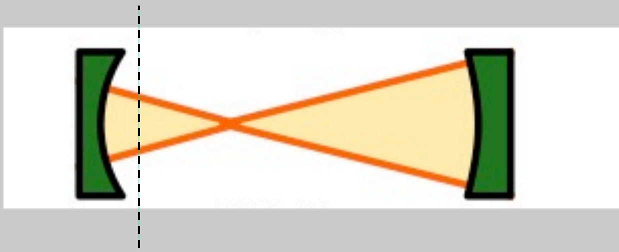
- Easily identified stable resonators are actually at edge of stabilty





Determining beam size, radius of curvature

- From *q* parameter
  - For stable mode: $$q_0 = \frac{(A-D)}{2C} \pm \frac{1}{2C}\sqrt{(A-D)^2 + 4BC}$$
  - And $$\frac{1}{q_0} = \frac{1}{R} - i\frac{\lambda}{\pi w^2}$$ Beam waist is where Re[1/q_0]=0
  - So $$w^2 = -\frac{\lambda}{\pi \operatorname{Im}\left[q_0^{-1}\right]}$$



  - Which *w* is this? It is at the start/end position of the round trip ABCD system matrix
  - Local beam w and R from q:

$$\text{beamWQ}(n\_,\text{qinv}\_) := \sqrt{-\frac{\lambda 0}{\pi\, n \operatorname{Im}(\text{qinv})}}$$

$$\text{beamRQ}(n\_,\text{qinv}\_) := \frac{1}{\operatorname{Re}(\text{qinv})}$$

Finding waist at other location

- Given local w1 and R1, we can solve to find beam waist size (w0) and location (z):

$$R_1 = z\left(1+\frac{z_R^2}{z^2}\right) \qquad w_1 = w_0\sqrt{1+\frac{z^2}{z_R^2}} \qquad z_R = \frac{n\pi w_0^2}{\lambda_0}$$

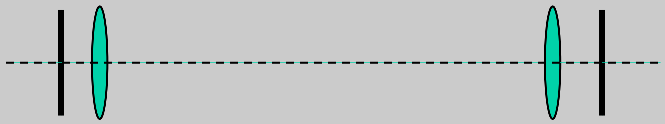
  – Eliminate zR, solve for z, then solve for w0

$$\text{zWaist}[\text{w1\_},\text{r1\_},\text{n\_}] := \frac{\pi^2\,\text{r1}\,\text{w1}^4}{\text{r1}^2\left(\frac{\lambda 0}{n}\right)^2 + \pi^2\,\text{w1}^4}$$

$$\text{w0Waist}[\text{w1\_},\text{r1\_},\text{n\_}] := \frac{\lambda 0\,\text{w1}\,|\text{r1}|}{n\sqrt{\text{r1}^2\left(\frac{\lambda 0}{n}\right)^2 + \pi^2\,\text{w1}^4}}$$

- Trick for cavity arrangement

For curved end mirror, split:



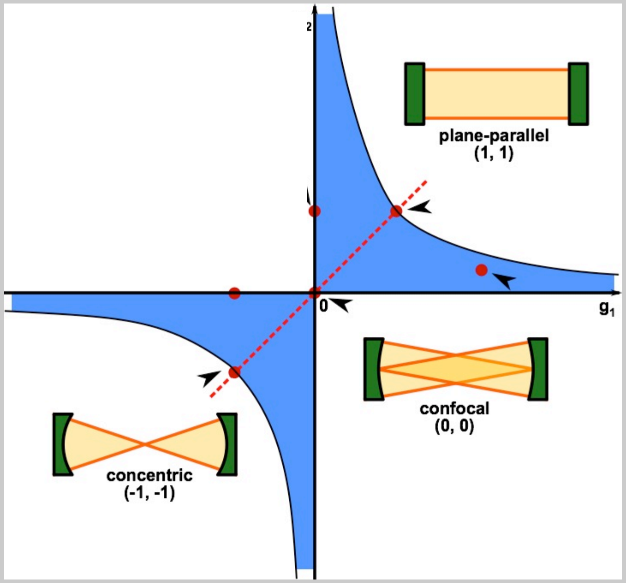
Then mode is collimated at end.

Symmetric cavities (R_1=R_2)

- At end mirror, wavefront curvature matches surface of mirror.
  - Plano end mirror: waist at mirror
  - Symmetric cavity (R_1=R_2, g_1=g_2): waist location at center. Can fully specify mode w/o ABCD.
- Use Gaussian beam equations:

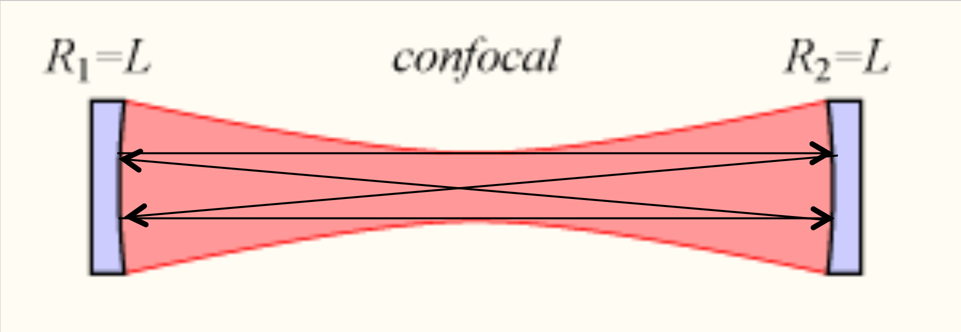
$$R = z\left(1+\frac{z_R^2}{z^2}\right) \rightarrow \frac{L}{2}\left(1+\frac{4z_R^2}{L^2}\right)$$

$$z_R = \frac{L}{2}\sqrt{\frac{2R}{L}-1} \qquad w_0 = \sqrt{\frac{\lambda L}{2\pi}\sqrt{\frac{2R}{L}-1}}$$



Confocal cavity

- Symmetric cavity, focal points overlap



  - Cavity length is equal to the confocal parameter

$$L = 2z_R = b$$

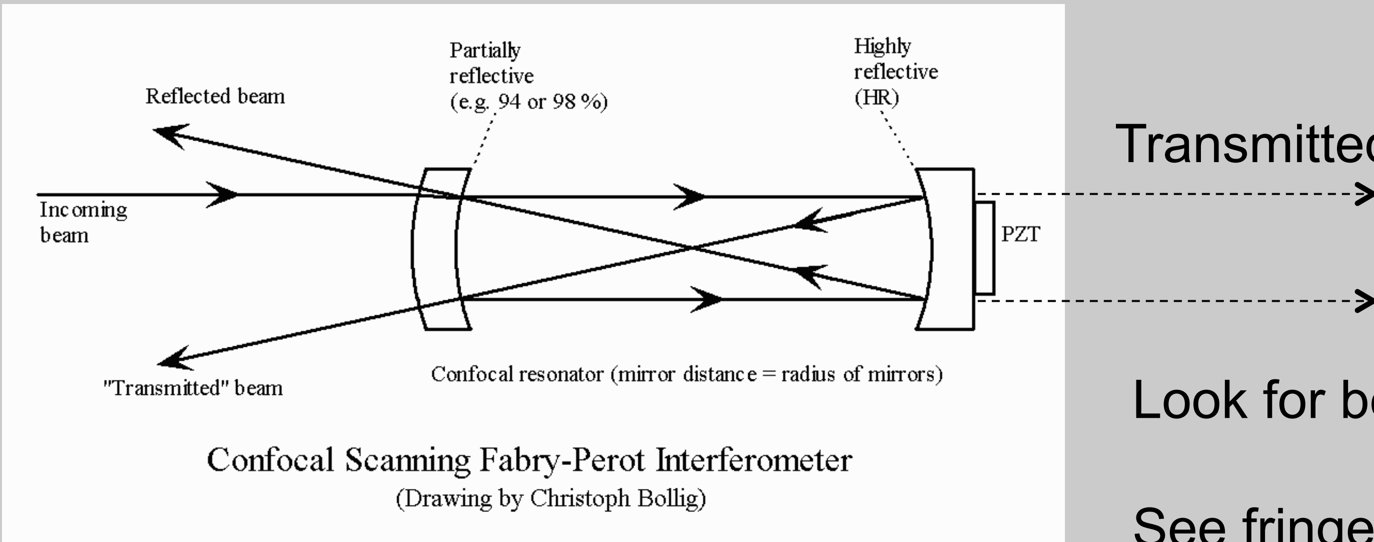
  - Spot size:

$$w_0 = \sqrt{\frac{\lambda L}{2\pi}}$$

  - Confocal cavity has only ~40% variation of mode size along cavity
  - Least sensitivity to angular misalignment.

Scanning Fabry-Perot interferometer

- Confocal resonator



Confocal Scanning Fabry-Perot Interferometer
(Drawing by Christoph Bollig)

Transmitted beams

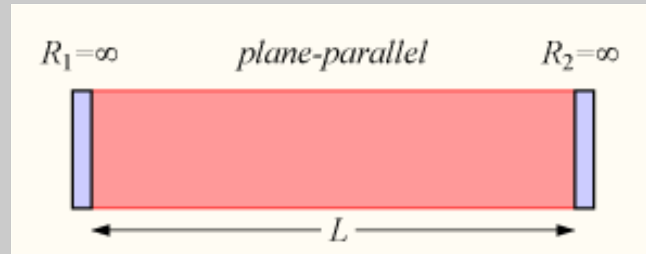
Look for beam overlap

See fringes: transmission through curved mirrors makes beams diverge

- Mode-matching: make input beam identical to desired output beam
  - Set initial beam size and focusing lens
  - Focus in center of cavity

Near-planar and concentric limits

- Near-planar: R very large, >> L



$$z_R = \frac{L}{2}\sqrt{\frac{2R}{L}-1} = \frac{L}{2}\frac{2R}{L}\sqrt{1-\frac{L}{2R}} \approx R\left(1-\frac{L}{4R}\right)$$

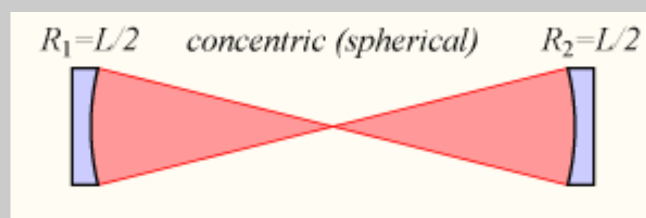
  – Large, constant mode size. sensitive to angle misalignment

- Near-concentric: L ~ 2R
  – Let L = 2R – δL

$$z_R = \frac{L}{2}\sqrt{\frac{2R}{L}-1} = \frac{2R-\delta L}{2}\sqrt{\frac{2R}{2R-\delta L}-1} \approx R\sqrt{\left(1+\frac{\delta L}{2R}\right)-1} \approx \sqrt{\frac{R\delta L}{2}}$$

  – Small mode in center, large mode at curved mirrors

In general, position on stability map controls mode size throughout cavity.



Higher-order resonator modes

– Higher-order resonator modes follow the Hermite-Gaussian (or Laguerre-Gaussian) funcitons

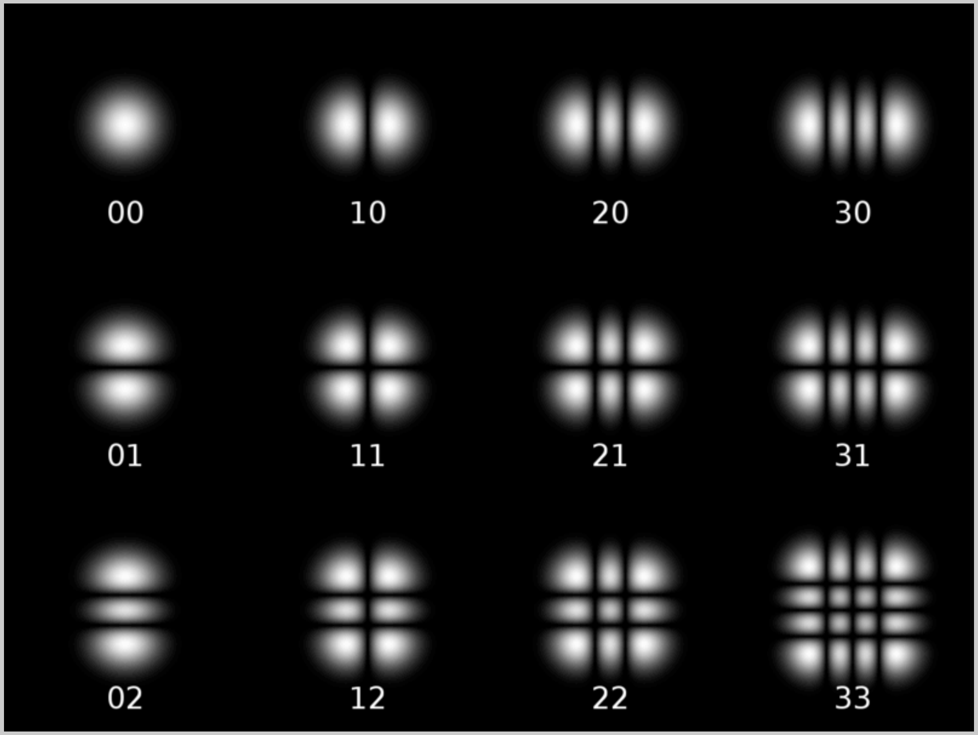
$$E(x,y,z)=A_0 e^{-i(kz-\eta_{lm}(z))}\frac{w_0}{w(z)}e^{-\frac{x^2+y^2}{w^2(z)}}H_l\left(\frac{\sqrt{2}x}{w(z)}\right)H_m\left(\frac{\sqrt{2}y}{w(z)}\right)e^{-i\frac{k(x^2+y^2)}{2R(z)}}$$

$$\eta_{lm}=(1+l+m)\tan^{-1}\left(\frac{z}{z_R}\right)$$

R(z) is independent of mode order

Resonant frequencies depend on mode indices.

Extent of field is larger as mode index increases – more diffraction loss.



Eigenvalues for high-order standing waves

- High-order modes generally have different resonant frequencies (*n* is longitudinal mode index)

$$\nu_{nlm} = \frac{c}{2L}\left(n + \left(\frac{1+l+m}{\pi}\right)\cos^{-1}\left(\pm\sqrt{AD}\right)\right)$$

– 2 mirror resonator:

$$\nu_{nlm} = \frac{c}{2L}\left(n + \left(\frac{1+l+m}{\pi}\right)\cos^{-1}\left(\pm\sqrt{g_1 g_2}\right)\right)$$

+ if g_1 and $g_2 > 0$

- if g_1 and $g_2 < 0$

– Confocal: $g_1 = g_2 = 0$

$$\nu_{nlm} = \frac{c}{4L}\left(2n + \left(1+l+m\right)\right)$$

Even modes are degenerate
Odd modes degenerate
Offset by c/4L

Mode spacing in resonators

- Lowest mode:

$$\nu_{nlm} = \frac{c}{2L}\left(n + \left(\frac{1+l+m}{\pi}\right)\cos^{-1}\left(\pm\sqrt{g_1 g_2}\right)\right) \rightarrow \frac{c}{2L}\left(n + \frac{1}{\pi}\cos^{-1}\left(\pm\sqrt{g_1 g_2}\right)\right)$$

  - The cos-1( ) factor is constant, so $\Delta\nu = \frac{c}{2L}$
- Multi-mode resonator
  - For a stable resonator $0 \le g_1 g_2 \le 1 \rightarrow \frac{\pi}{2} \ge \cos^{-1}\left(\sqrt{g_1 g_2}\right) \ge 0$
  - High-order modes will generally lie between peaks for the lowest order mode

Example: 1.5GHz FP

- Free spectral range = 1.5GHz
  - For a well-aligned confocal FP, all even modes are degenerate
  - odd modes are midway between TEM00 mode frequencies, so stated FSR is $$\Delta\nu = \frac{c}{4L} \rightarrow L = \frac{c}{4\Delta\nu}$$
  - Cavity length L = 5.0cm
  - Mode waist radius: $$w_0 = \sqrt{\frac{\lambda L}{2\pi}}$$
    w_0 ~ 71um (for 632.8nm)
  - Output mode waist radius: $\sqrt{2}w_0 = 100\text{um}$
  - For a general resonator, the resonant frequency is different for higher-order modes.

Resonator stability analysis

- Resonators are designed under different constraints and can be optimized
- Plot a stability parameter to show stable zone(s) of operation
  - Stability condition: $-1 < \frac{A+D}{2} < 1$
  - By convention to plot *s* parameter:

$$s = 1 - \left(\frac{A+D}{2}\right)^2$$

Parameter is always positive in stable zone

Focusing resonator

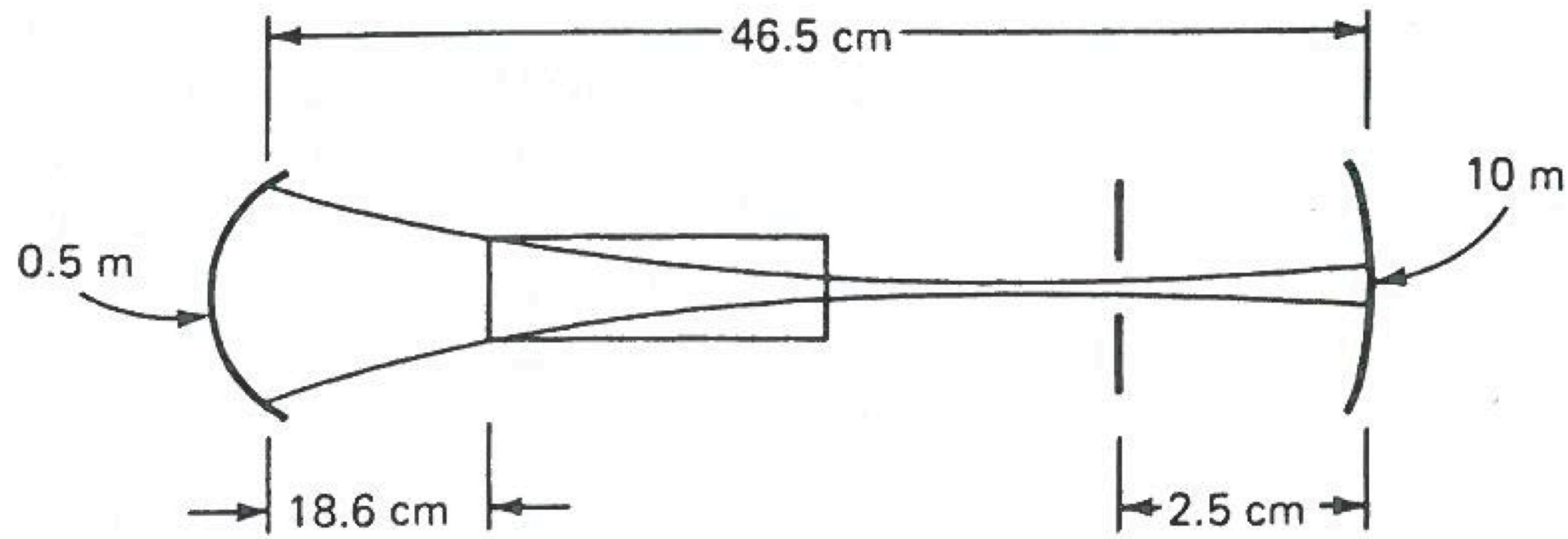


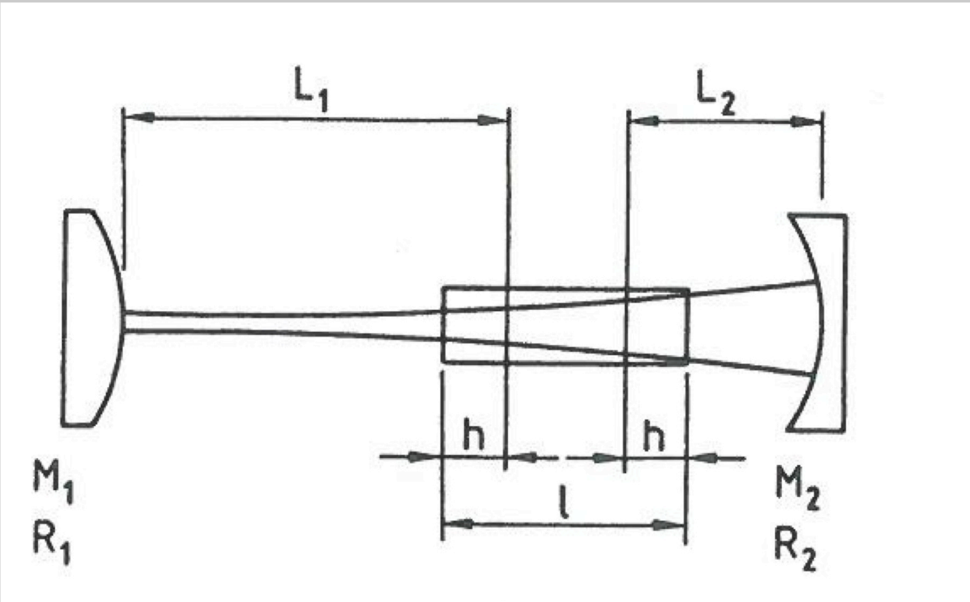
Fig. 5.20. Focusing resonator geometry

Koechner “Solid-state laser engineering”

Nearly hemispherical resonator

- large mode on left
- Laser rod acts as aperture to limit TEM00 operation
- Second aperture to clean up beam

Convex-concave resonator



Koechner "Solid-state laser engineering"

Shorten cavity by using a convex end mirror

Weak thermal lensing in rod

- Small spot on convex mirror
- Too intense for pulsed operation

Internal telescope resonators

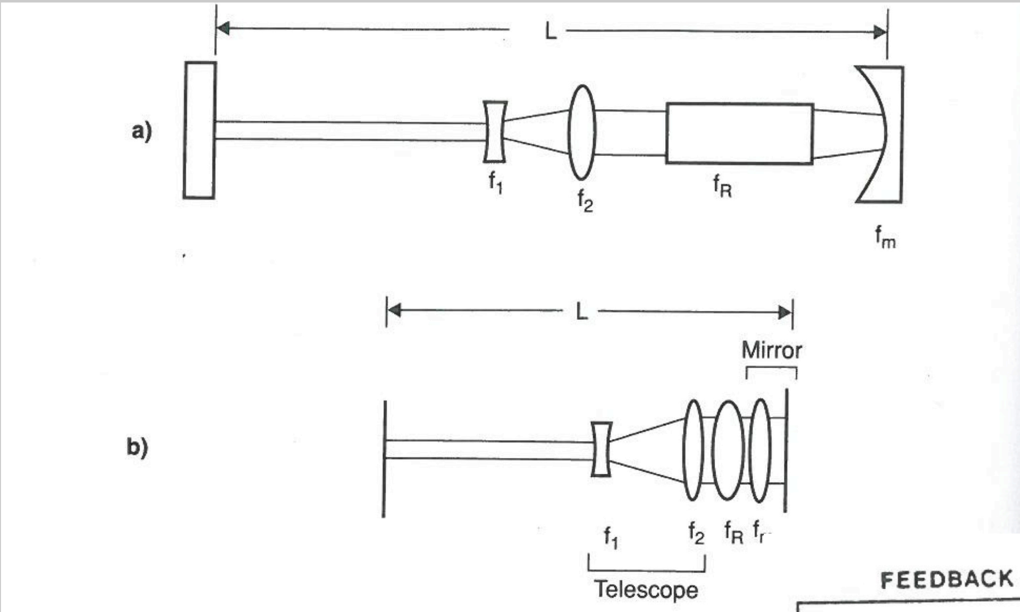
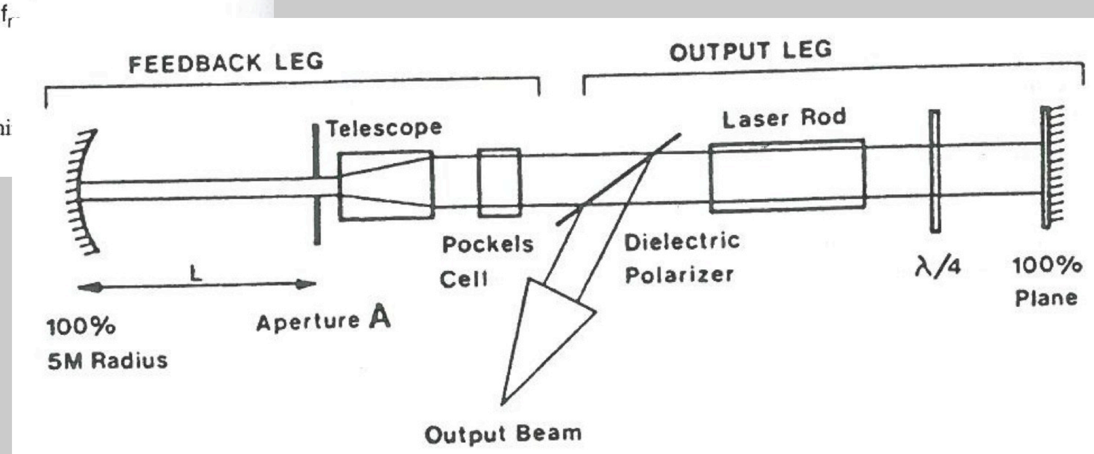


Fig. 5.27. Resonator with internal telescope, focusing laser rod, and curved mi
optical equivalent (b)

Alternatives to expand mode in gain medium while using a short resonator.



Koechner “Solid-state laser engineering”

Mechanically-stable resonator design

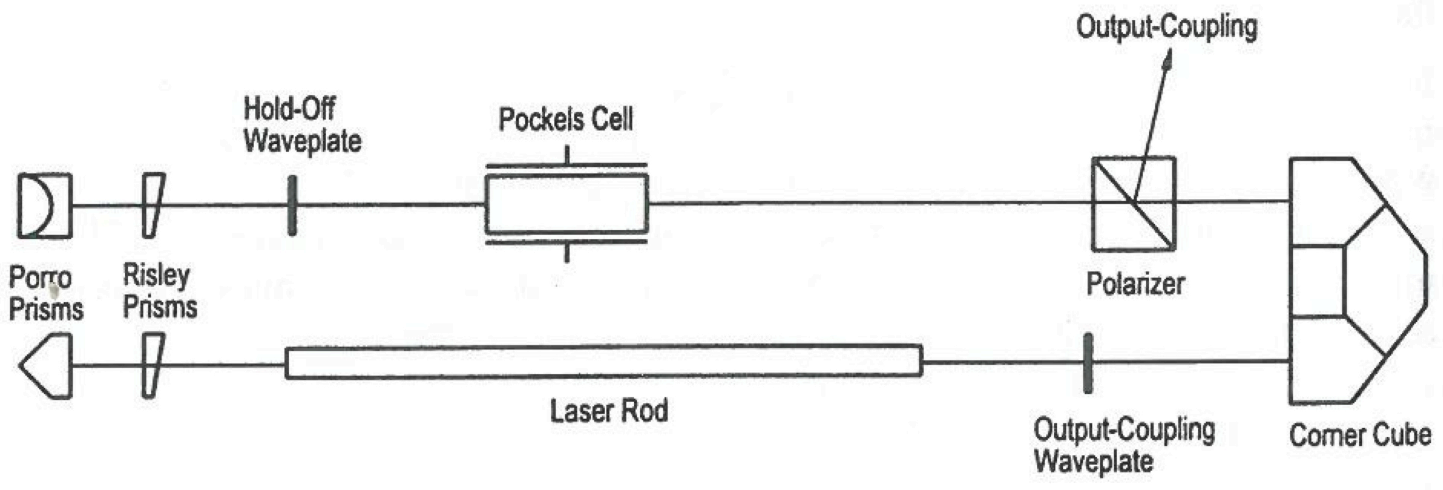


Fig. 5.30. Resonator using cross Porro prisms and polarization output coupling

Koechner "Solid-state laser engineering"

Corner cube returns beam parallel but with lateral offset.
Porro prism does the same but only in one direction.
Risley prisms are wedges that can be rotated to steer beams.

Zig-zag slab resonator

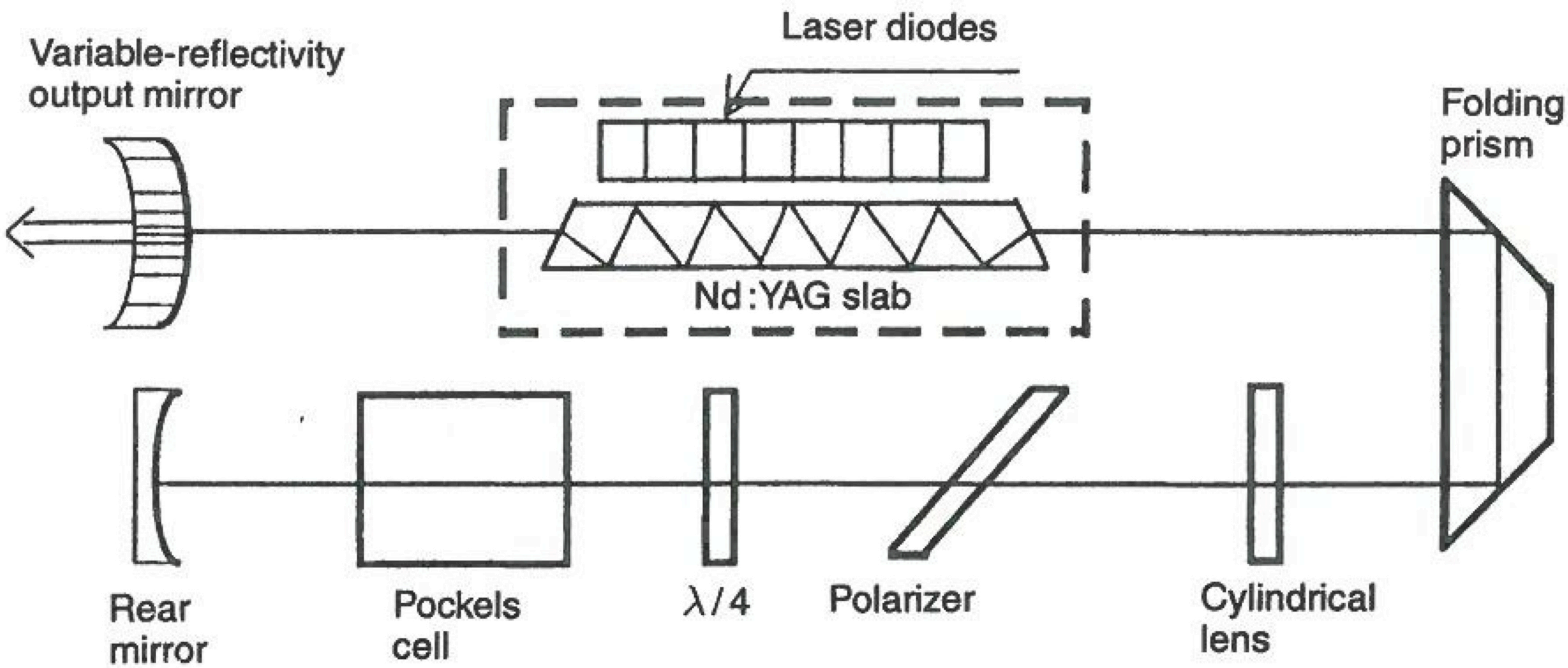


Fig. 5.50. Diode-pumped Nd:YAG slab laser with positive-branch unstable resonator and variable reflectivity output coupler [5.76]

Koechner "Solid-state laser engineering"

Astigmatic compensation

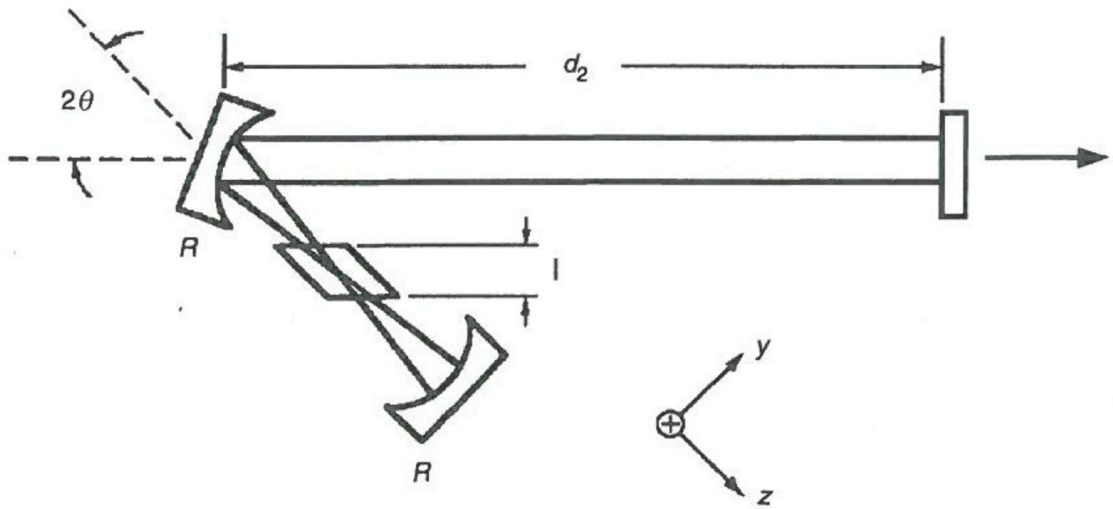


Fig. 5.29. Astigmatic compensation of a folded resonator containing an optical element at Brewster's angle

Koechner "Solid-state laser engineering"

- Entering a tilted interface, beam propagating inside the crystal is wider
- The wider beam has a longer Rayleigh range and also travels farther to the beam waist – this leads to astigmatism
- Use tilted mirrors to compensate the astigmatism.