4-16-08

Note Title 4/16/2008

Sample probs from ch. 4

* What does it mean for observables to be compatible? Why is this important?

* Prove that $\frac{d}{dt}\langle \vec{p} \rangle = \langle -\nabla V \rangle$

* Suppose $f(\vec{r}) = f(r)$. Solve $\nabla^2 f(r) = 0$ by directly integrating the equation. I will give you

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

* What are the Eigenfunctions of the angular part of $\nabla^2$ on the unit sphere.

* For an infinitely hard sphere

$$V(r) = \begin{cases} \infty & \text{if } r \le a \\ 0 & \text{if } r > a \end{cases}$$

The Schrodinger Eqn reduces to

$$\frac{d^2 u}{dr^2} = \left[\frac{\ell(\ell+1)}{r^2} - k^2\right] u \qquad u = rR(r)$$

The general solution is

$$u(r) = A\, r\, j_\ell(kr) + B\, r\, N_\ell(kr)$$

where $k = \frac{\sqrt{2mE}}{\hbar}$

Compute $R(r)$ for $r \geq a$ when $\ell = 0$. Compute $E_n$.

A particle in a finite spherical well sees the following potential

$$V(r) = \begin{cases} -V_0 & \text{if } r \leq a \\ 0 & \text{if } r > a \end{cases}$$

Find the form of the solution to the $\ell=0$ Schrodinger.
i.e. Solve the TISE up to the point of applying the BC.
State what the BC are, but don't apply them.

I want to see explicit express. for the 2 B.C.

* For the hydrogen atom
  1) write down the coulomb potential
  2) write down the "centrifugal" term of the TISE.

  3) For what value of $r$ are these balanced? i.e. equal in magnitude but opposite in sign.

  4) prove that the $r \to \infty$ behavior of solutions to the TISE must be exponentially decaying.

* Compute the wavelength and frequency of a photon sufficiently energetic to ionize an electron in the state $\psi_{100}$

* explain why we would try to measure $L^2$ and $L_z$ rather that $L_x$, $L_y$ & $L_z$.

* IF $L_z f = \mu f$ and the raising operator for $L_z$ is $L_x + i L_y$, explain why

there must exist a eigenstate of $L_z$ (call it $f_t$) such that

$L_+ f_t = 0$.

* in spherical coordinates

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

if $\vec{r} = r\hat{r}$ then compute the operator for angular momentum $\underline{L}$.

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See the ch. 3 problems from Sunday's review. plus ...

* Let $\hat{Q}$ be an operator represented by the matrix

$$Q = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$ compute the

normalized eigenvectors of $Q$ ($|e_1\rangle$, $|e_2\rangle$) and $\mathcal{E}$-values $q_1, q_2$.

Show explicitly that

$$\hat{Q} = q_1 |e_1\rangle\langle e_1| + q_2 |e_2\rangle\langle e_2|$$