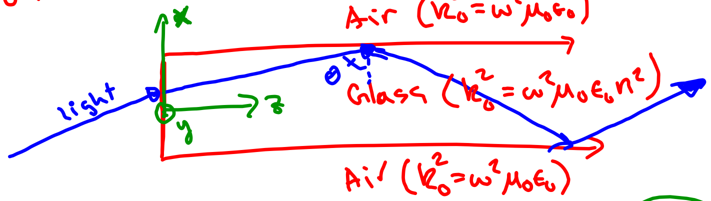
Reading Today: Chillwell Sec. 3 & 4.
Tomorrow: ??

Waveguides. We already talked about wave-guide in a metal casing.

Another kind of waveguide, sort of hinges on total internal reflection.



for plane waves $\frac{\omega}{k_0}=\frac{c}{n}$ $k_0=\frac{\omega n}{c}=n\omega\sqrt{\mu_0\epsilon_0}$, with $n=\sqrt{\mu_r\epsilon_r}$

$k_0=\omega\sqrt{\mu\epsilon}$

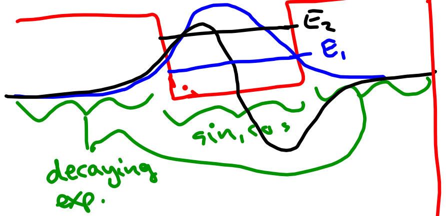
General solution to the wave eqn in 2-D {no y-dependence} is (TE wave)

$H_z=\hat{A}e^{ik_z z}e^{ik_x x}$; $k_x^2+k_z^2=k_0^2=\omega^2\mu\epsilon$

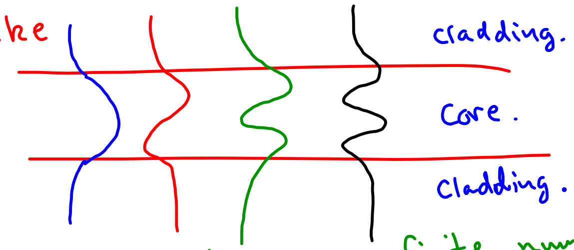
Remind ourselves of quantum (finite square well

$E_\psi = V + K$

K determines by $\frac{d^2\psi}{dx^2}$



So what do mode profiles look like



There are always a finite number of propagating bound modes because k_x has to be imaginary in the cladding (for exponential decay) and real in the cor (for oscillatory).

Last class we defined $\beta = n\sin\theta$

$k_z = k_0 \sin\theta \qquad \frac{k_z}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{k_0}{\omega\sqrt{\mu_0\epsilon_0}}\sin\theta = n\sin\theta \equiv \beta$

$$\Rightarrow \frac{k_z}{\omega\sqrt{\mu_0\epsilon_0}} = \beta$$

From befor $\left(k_x^2 + k_z^2 = k_0^2\right) / \left[\omega^2\mu_0\epsilon_0\right]$

$$\underbrace{\frac{k_x^2}{\omega^2\mu_0\epsilon_0}} + \beta^2 = n^2$$

is actually α^2 from the last class.

$$\alpha^2 + \beta^2 = n^2$$

So, from all my arguing, what's the condition for β for waveguide modes in terms of $n_{cladding}$, n_{core}?

$$n_{cladding} < \beta < n_{core}$$

effective index of the mode.

Application:

For a simple slab wave guide, you can solve like finite square well prob.

For multi-layer problems (or single layer) we can use our matrix.

Argument, α is imaginary in cover and substrate but in order for decay rather than growth, both waves must be outgoing.

$$\begin{pmatrix} u^+ \\ v^+ \end{pmatrix} = \begin{pmatrix} 1 \\ \gamma \end{pmatrix} u^+ \quad ; \quad \begin{pmatrix} u^- \\ v^- \end{pmatrix} = \begin{pmatrix} 1 \\ -\gamma \end{pmatrix} u^-$$

Also we know $\begin{pmatrix} u_c \\ v_c \end{pmatrix} = M \begin{pmatrix} u_s \\ v_s \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 1 \\ \gamma_c \end{pmatrix} u_c = M \begin{pmatrix} 1 \\ -\gamma_s \end{pmatrix} u_s$$

$$u_c = (m_{11} - \gamma_s m_{12}) u_s \ ; \ \gamma_c u_c = (m_{21} - \gamma_s m_{22}) u_s$$

$$\gamma_c (m_{11} - \gamma_s m_{12}) \cancel{u_s} = (m_{21} - \gamma_s m_{22}) \cancel{u_s}$$

1 $$\boxed{\gamma_c m_{11} - \gamma_c \gamma_s m_{12} - m_{21} + \gamma_s m_{22} = 0}$$

modal dispersion function

Gist of the whole lecture

- waveguides are made up of a high index core and low index cladding.
- You get a finite discrete number of propagating bound modes with $n_{cladding} < \beta$, $\beta < n_{core}$.
- You can find the modes by finding the zeros of eqn. 1.