Unstable resonators and generalied ABCD matrices

Unstable resonators

- negative and positive branch

Generalized ABCD matrices

- Gaussian mirror
- Gaussian gain distribution
- parabolic index guiding
- gain guiding

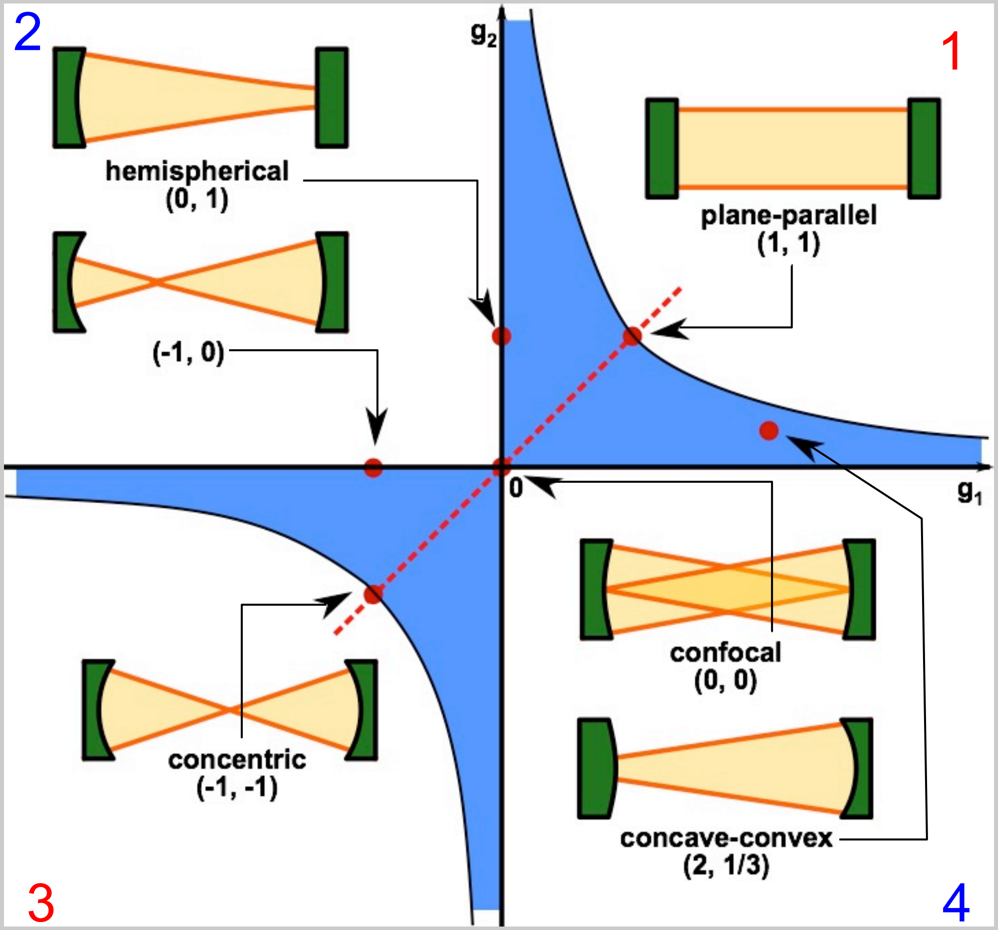
Unstable resonators

- Cavity is stable if $-1 < \frac{A+D}{2} < 1$ $\qquad -1 < 2g_1 g_2 - 1 < 1$

Stable in shaded regions
Unstable in white regions

$$0 \le g_1 g_2 \le 1$$

$$g_1 = 1 - \frac{L}{R_1} \qquad g_2 = 1 - \frac{L}{R_2}$$



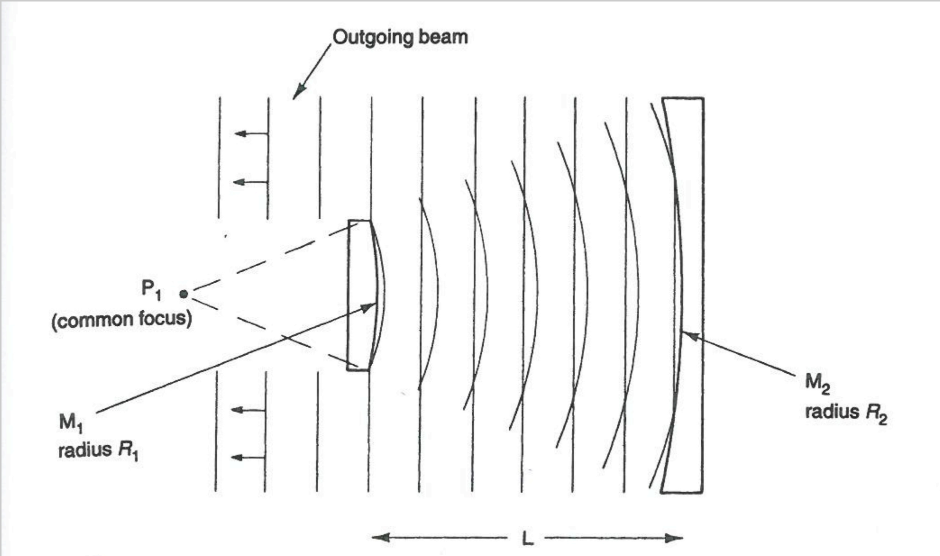
1st and 3rd quadrants:
Positive branch:
$0 < g_1 g_2 < 1$ **stable**
$g_1 g_2 > 1$ **unstable**
For unstable resonator:
No focal point inside resonator

2nd and 4th quadrants:
Negative branch: $g_1 g_2 < 0$
One center of curvature
inside resonator
For unstable resonator: focal point inside resonator

Unstable resonators

Unstable resonators often use beam magnification to output couple past a mirror. Diffraction losses are the means of output coupling.

- Gain must be sufficient to overcome diffractive losses.



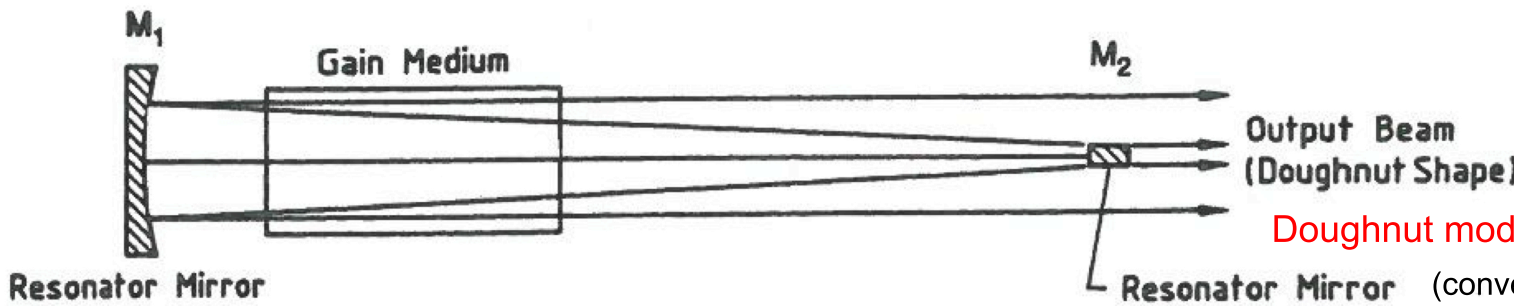
Telescope magnification:

$$M = \frac{f_2}{f_1} = \frac{R_2}{R_1}$$

Output coupling loss per round trip:

$$\frac{A_2}{A_1} = M^2 = \left(\frac{R_2}{R_1}\right)^2$$

Positive branch: no internal focus in resonator

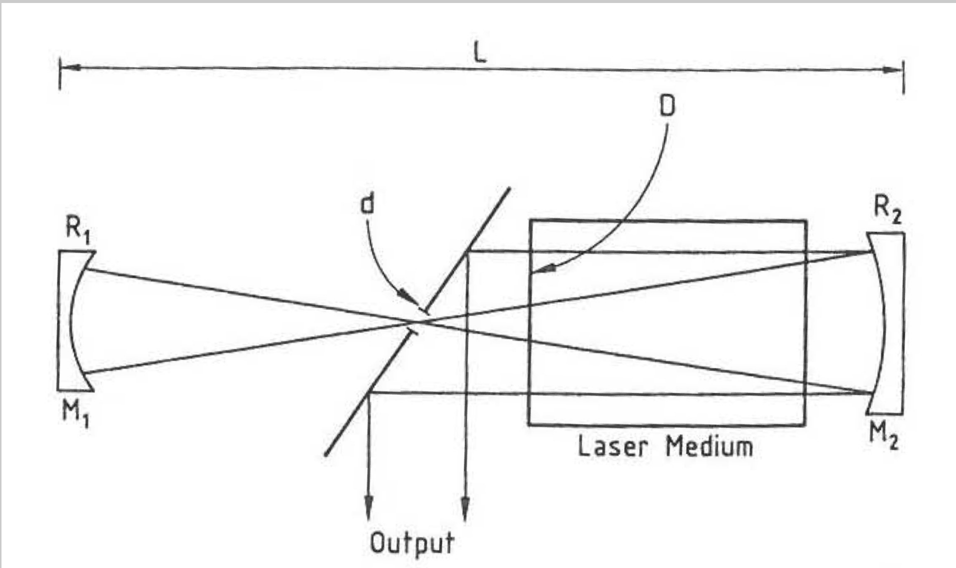


Doughnut mode still focuses well

(convex)

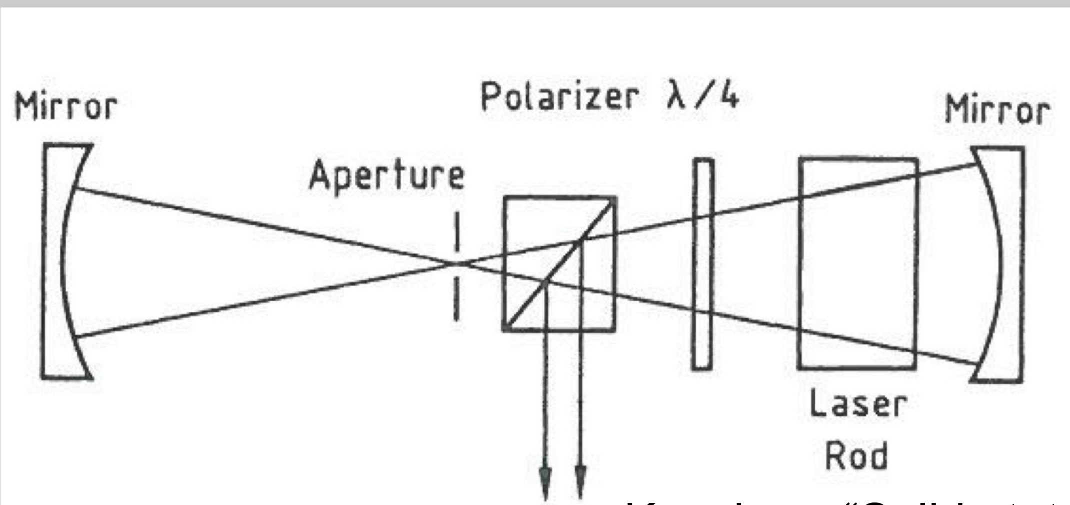
Koechner "Solid-state laser engineering"

Negative branch unstable resonators



Aperture at intermediate focus
- Acts as an internal spatial filter

"scraper mirror" output



Polarization-coupled output

Koechner "Solid-state laser engineering"

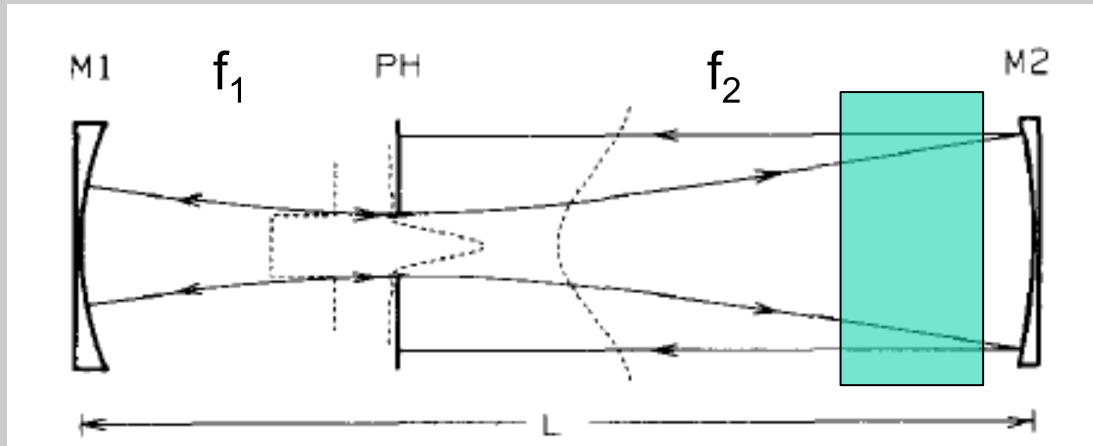
Self-filtering unstable resonator

Gobbi, Opt Commun 52, 195 (1984)

- Confocal resonator with magnification

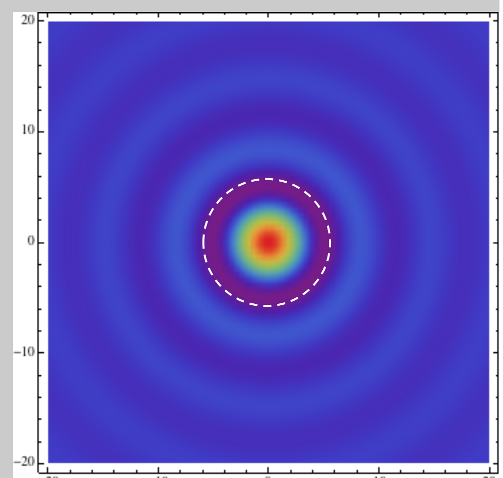
sequence:

1. Collimated beam from M2 toward PH
2. PH clips beam, reduces energy by ratio f2/f1
3. Airy diffraction pattern imaged to PH by M1



$$E(r) \propto J_1\left(\frac{ka}{f_1}r\right) \Big/ \frac{ka}{f_1}r$$

first zero at $r = 1.22 f_1 \lambda / 2a$



4. PH radius at first zero: passes 84% of power
5. M2 recollimates beam

For M=3, round trip transmission ~ 30%
Use with high gain

Generalized ABCD

- Examples:
  - Variable output coupling mirrors
  - Radially-dependent gain
  - Parabolic refractive index profiles
  - Parabolic gain profiles – gain guiding
- ABCD with gain and loss lead to complex terms
  - Qualitative change to stability
  - Need additional modeling to calculate net gain and loss (ABCD is for beam shape, not amplitude)

Variable reflectivity mirror

- Gaussian mirror: graded reflectivity dielectric coating
  - Beam curvature unaffected
  - Beam size is reduced:

$$e^{-r^2/w_2^2} = e^{-r^2/w_1^2} e^{-r^2/w_m^2} \qquad \frac{1}{q_2} = \frac{1}{R} - i\frac{\lambda}{\pi}\left(\frac{1}{w_1{}^2} + \frac{1}{w_m{}^2}\right) = \frac{1}{q_1} - i\frac{\lambda}{\pi w_m{}^2}$$

Compare to gaussian beam ABCD

$$q_2{}^{-1} = \frac{C + Dq_1{}^{-1}}{A + Bq_1{}^{-1}}$$

$$A = 1, B = 0$$

$$C = -i\frac{\lambda}{\pi w_m^2}, D = 1$$

Gaussian mirror

$$\begin{pmatrix} 1 & 0 \\ -i\frac{\lambda}{\pi w_m^2} & 1 \end{pmatrix}$$

lens

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Acts like a lens with imaginary focal length!

Spatial gain narrowing

- Assume longitudinal pumping with Gaussian beam
- Even though gain adds to pulse energy, effect is similar to the Gaussian mirror

$$e^{-r^2/w_2^2} = e^{-r^2/w_1^2} \exp\left[-\frac{\Gamma_{stor}}{\Gamma_{sat}} \exp\left[-2r^2 / w_g^2\right]\right]$$

$$\approx G_0 e^{-r^2/w_1^2} \exp\left[-2r^2 / w_g^2\right]$$

G0 is peak gain on axis
Expand exp[ ] in exponent, keeping parabolic term

$$\begin{pmatrix} 1 & 0 \\ -i\dfrac{2\lambda}{\pi w_g^2} & 1 \end{pmatrix}$$

ABCD only keeps track of beam width and radius of curvature – not loss or gain

Both versions enforce “stability”
Need to be careful about results of trace.

Gradient index profiles

- Laser rod has extended interaction with beam
  - Thermal lensing and gain affect beam propagation
- Ideal lens changes wavefront curvature

$$E_{out}(r) = E_{in}(r)e^{-ikr^2/2f}$$

  - Can accomplish the same effect with a gradient index medium, e.g.

$$n(r) = n_0\left(1 - \frac{k_2}{2k}r^2\right)$$

k_2 is a constant to control curvature

  - For a thin medium:

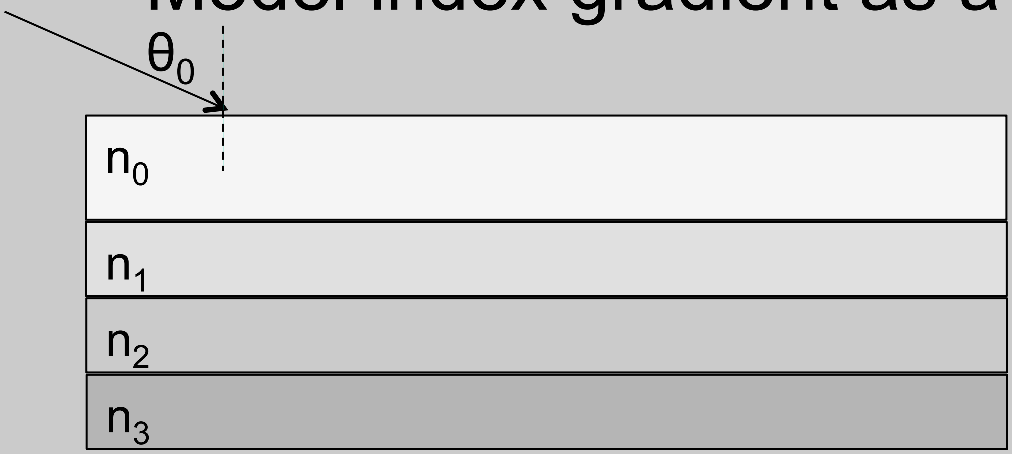
$$E_{out}(r) = E_{in}(r)e^{ikn_0\left(1-\frac{k_2}{2k}r^2\right)L} = E_{in}(r)e^{ikn_0L}e^{-ikL\frac{n_0k_2}{2k}r^2}$$

$$f = \frac{k}{n_0k_2L}$$

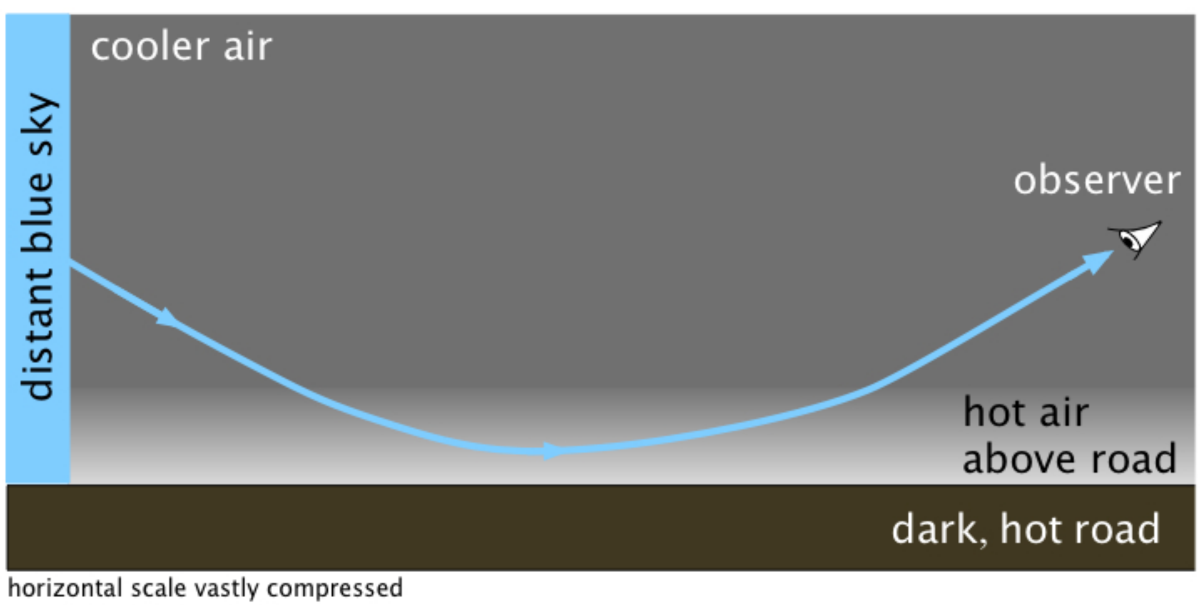
GRIN lens: diffuse ions into lens material
Thermal profiles: n[T(r)]

The mirage effect

- Model index gradient as a sequence of layers

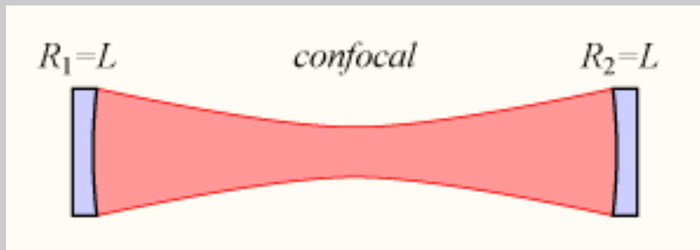


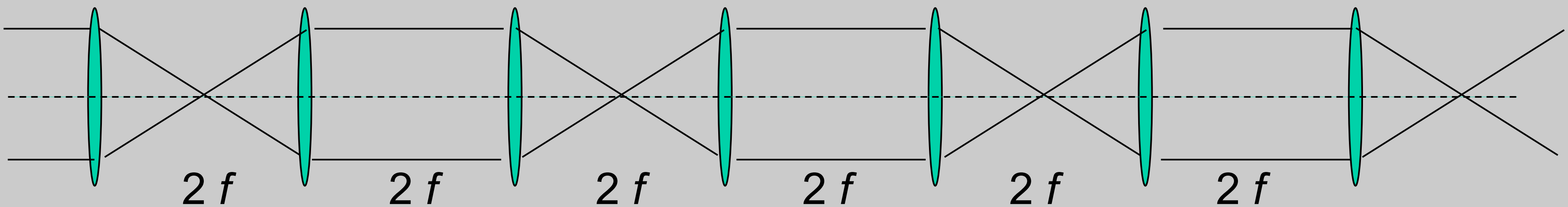
Find condition for turning point



Periodic lens model

- A resonator can be “unfolded” by modeling the curved mirrors as ideal lenses

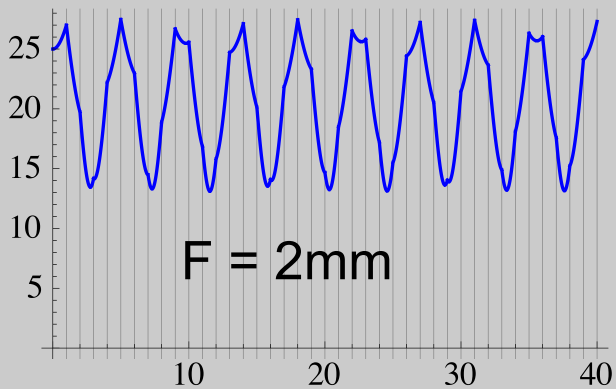


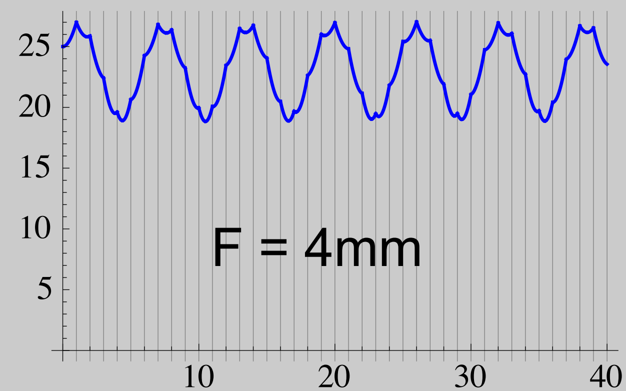


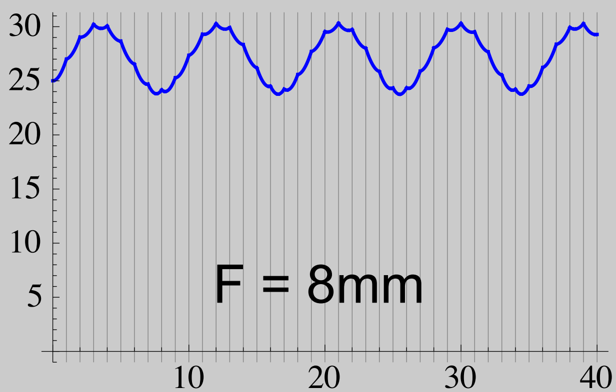
- Are there rays that will stay confined?
- If so, resonator is ***stable***.

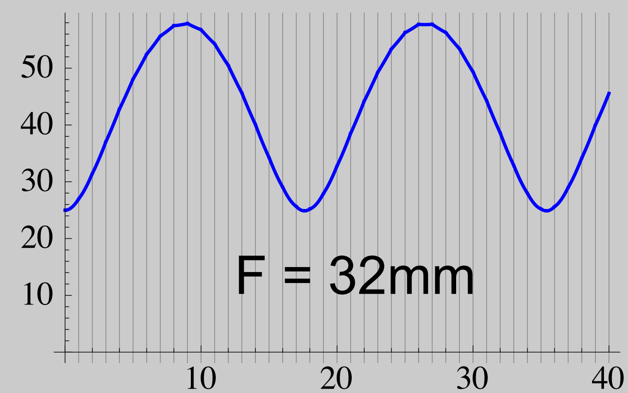
The lens waveguide

- A sequence of positive lenses can act as a waveguide









Weaker lens:
Smoother profile

Ray equation for parabolic gradient

- Parabolic index gradient: $n(r) = n_0\left(1 - \frac{k_2}{2k}r^2\right)$
- Ray equation: $$\frac{d^2 r}{dz^2} + \frac{k_2}{k} r = 0$$
- height and angle oscillate: compare to SHO

Solution, including initial conditions

$$r(z) = \cos(k_{osc} z) r_0 + \frac{1}{k_{osc}} \sin(k_{osc} z) r_0'$$

$$r'(z) = -k_{osc} \sin(k_{osc} z) r_0 + \cos(k_{osc} z) r_0'$$

$$k_{osc} = \sqrt{k_2 / k}$$

Note that the period of oscillation is $Z_{osc} = 2\pi\sqrt{k / k_2}$

Can put this into ABCD form:

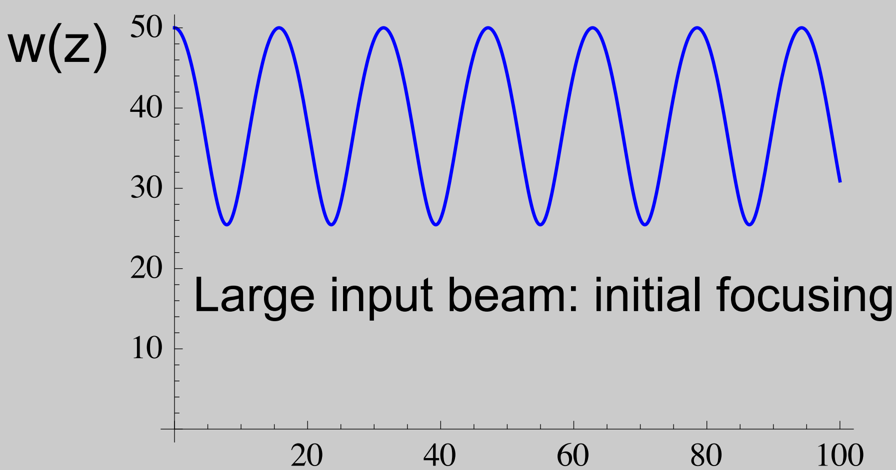
$$\begin{pmatrix} r(z) \\ r'(z) \end{pmatrix} = \begin{pmatrix} \cos(k_{osc} z) & \frac{1}{k_{osc}} \sin(k_{osc} z) \\ -k_{osc} \sin(k_{osc} z) & \cos(k_{osc} z) \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

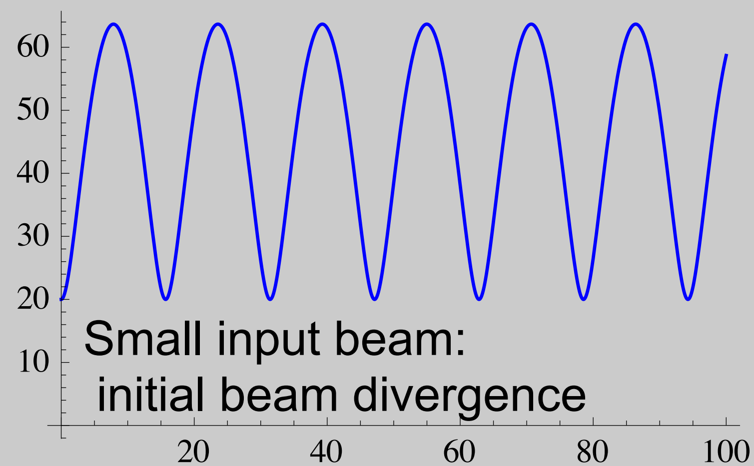
Gaussian beam solution

- Can use exact same ABCD matrix
  - Apply Gaussian ABCD rule:

$$q_1 = \frac{Aq_0 + B}{Cq_0 + D} \qquad q(z) = \frac{q_0 \cos(k_{osc} z) + \frac{1}{k_{osc}} \sin(k_{osc} z)}{-q_0 k_{osc} \sin(k_{osc} z) + \cos(k_{osc} z)}$$

  - Example: input beam waist, w_0 $\quad q_0 = -i\frac{\lambda}{\pi w_0^2}$





Gradient index waveguide

- Optical fibers: can be made with a gradient index
  - Is there a stable mode size?

$$q(z) = \frac{q_0 \cos(k_{osc} z) + \frac{1}{k_{osc}} \sin(k_{osc} z)}{-q_0 k_{osc} \sin(k_{osc} z) + \cos(k_{osc} z)} = q_0$$

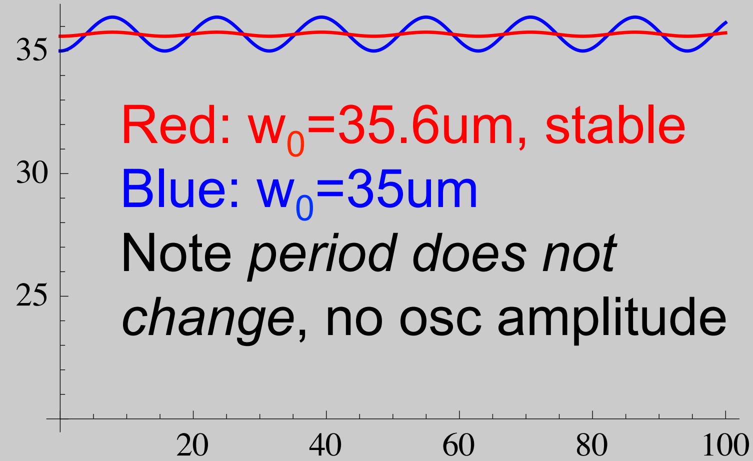
  - Solve for guided mode:

$$q_0 + \frac{1}{k_{osc}} \tan(k_{osc} z) = -q_0{}^2 k_{osc} \tan(k_{osc} z) + q_0 \qquad \rightarrow \left(q_0{}^2 k_{osc} + \frac{1}{k_{osc}} \right) \tan(k_{osc} z) = 0$$

For $\left(q_0{}^2 k_{osc} + \frac{1}{k_{osc}} \right) = 0$

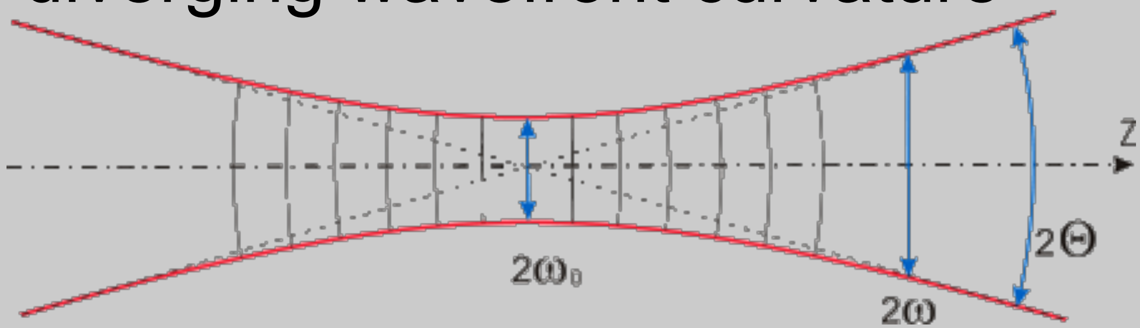
then no z-dependence

$$q_0{}^2 = -k_{osc}{}^2 \rightarrow z_R = 1/k_{osc} = \sqrt{k/k_2}$$



Guiding condition: wave perspective

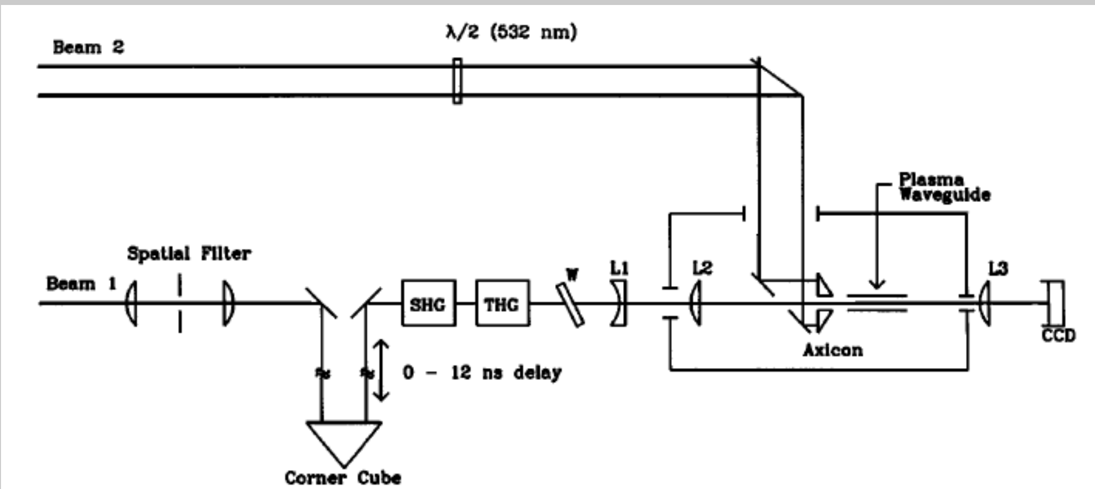
- w/o guiding, the beam will naturally develop diverging wavefront curvature

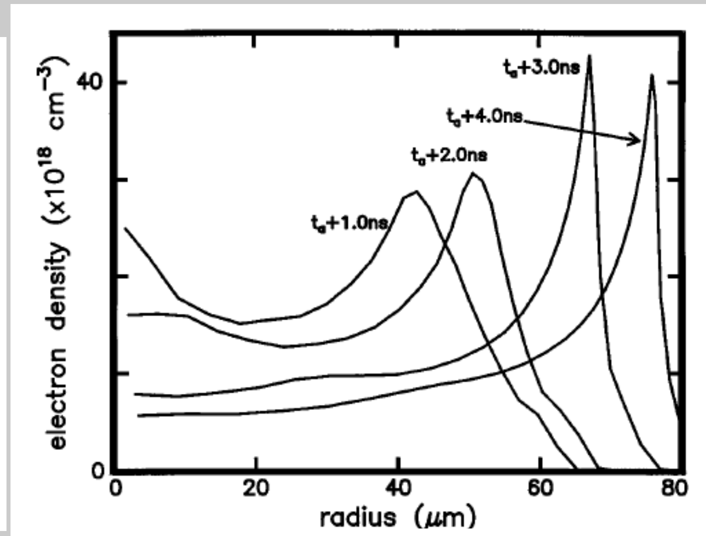


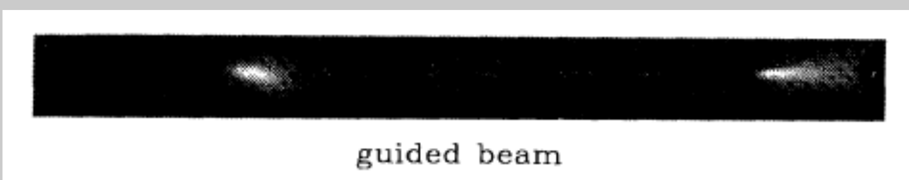
- Parabolic waveguide pulls central wavefront back, inducing focusing wavefront curvature
- If focusing balances diffraction: stable mode

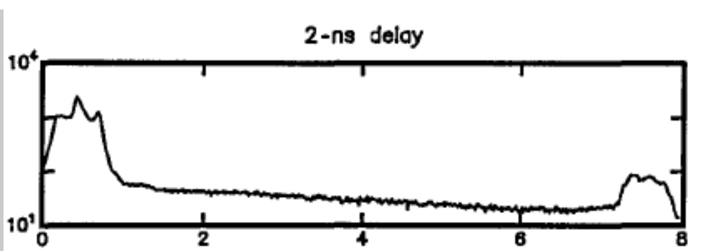
Example: plasma waveguide for intense pulses

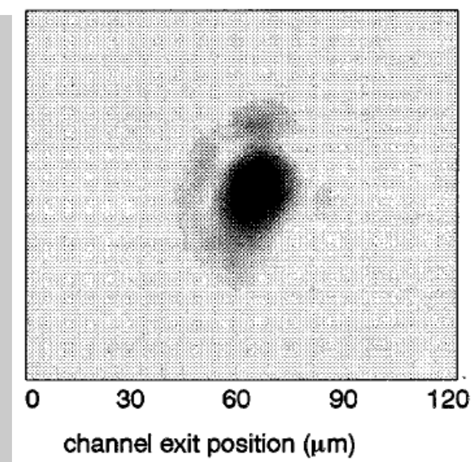
- Line focus in gas, ionization, radial expansion w/ shock wave





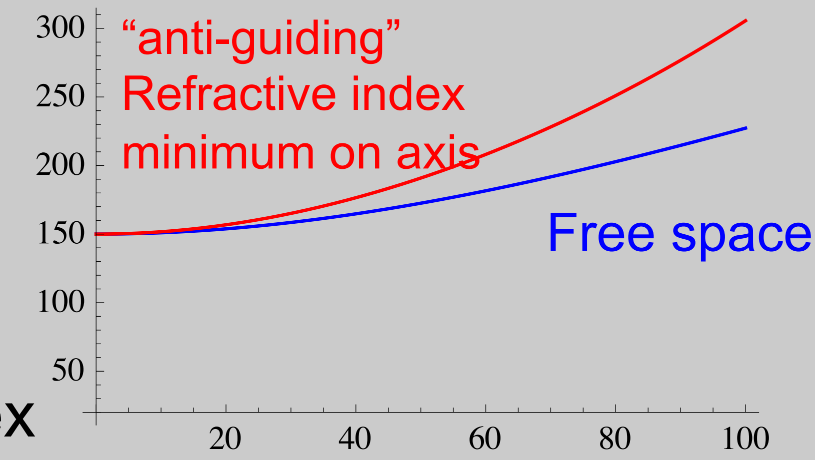






Generalizations for gradient index ABCD

- If $k_2 < 0$, refractive index parabola is inverted: cos[ ] to cosh[ ]
  - – Beam defocuses
- Gain guiding:
  - – Gain and loss are represented as complex index



$$n(r) = n_0\left(1 - i\frac{\alpha_2}{2k}r^2\right)$$

For e^{-ikz} convention, gain for $\alpha_2 < 0$

- Diffractive loss is compensated by gain along axis
- Guided mode has convex wavefronts
- Gain guiding leads to a breaking of rule: wavefront will not generally match end mirrors!