Matrices

Note Title 9/15/2006

$2.141 \in R^{1\times 1}$

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in R^{3\times 1}$ $[1, 2, 3] \in R^{1\times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nm} \end{bmatrix}, \quad (A)_{ij} = a_{ij}$$

$$[1\ 2\ 3]^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$(A^T)_{ij} = A_{ji}$$

$Ax = y \qquad A \in R^{n\times m}$

makes sense only if
$x \in R^m \quad y \in R^n$

$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\qquad x^T = [1, 2, 3]$

$\underline{Ax = y}$ $\qquad A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Linear comb. of columns of A $\qquad y = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$x_1 \begin{bmatrix} a_1 \end{bmatrix} + x_2 \begin{bmatrix} a_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Set of all possible linear combinations of the vector a_1, a_2 (i.e. the columns of A) is called the span of a_1, a_2

$$\text{span}\{\hat{x}, \hat{y}\} = R^2$$

So $Ax=y$

There exists a solution x iff x is in the span of the columns of A

$$A \in R^{N \times m} \qquad B \in R^{m \times p}$$

$$A \cdot B \in R^{N \times p}$$

$$(A \cdot B)_{ij} = \sum_{k=1}^{m} A_{im} B_{mj}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \\ \underline{2} & \underline{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \underline{2} \end{pmatrix}$$

$$\begin{aligned} x - y &= 0 \\ x + y &= 1 \\ 2x + 2y &= 2 \end{aligned} \Bigg\} \text{ redundant}$$

$$\begin{aligned} x - y &= 0 & \qquad x &= y \\ x + y &= 1 & \qquad x &= 1 - y \\ 2x + 2y &= 1 & \qquad x &= 1/2 - y \end{aligned}$$

$\text{span}\{\hat{x}, \hat{y}\} = R^2$

$c\hat{x} + d\hat{y}$ for all possible values of c, d

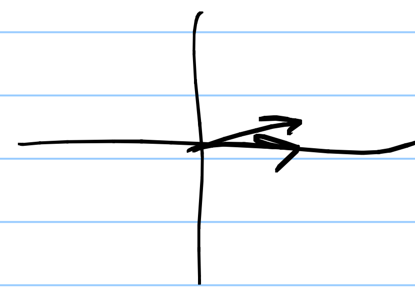
$\text{span}\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\} = R^1 = x\text{-axis}$

$\text{span}\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}27\\0\end{pmatrix}\right\} = x\text{-axis}$

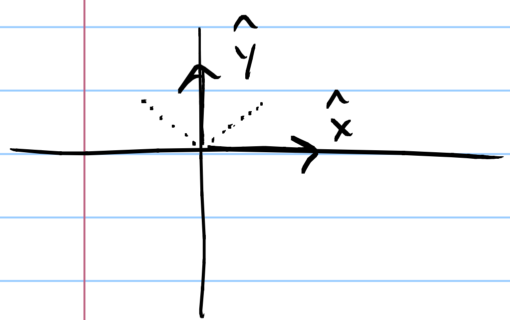
$\text{span}\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\} = R^2$ $\text{span}\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}\right\}$

Span

Dimensión



Basis vector



$\hat{x}=(1,0)$ $\hat{y}=(0,1)$

$\frac{1}{\sqrt{2}}(1,1)$ $\frac{1}{\sqrt{2}}(-1,1)$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$$