Lecture 38

Exam Wednesday April 23

Graded hmwks outside my door

Hmwk solns posted on the wiki

Shadowitz chapter 7

Learning objectives for exam 4 are that you will be able to:

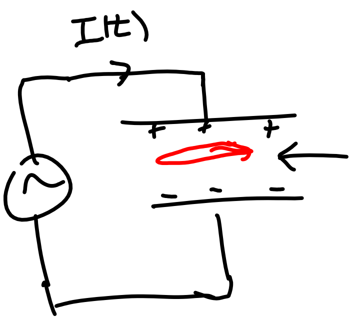
-be able to calculate the magnetic field due to a time changing electric field.
-be able to apply conservation of energy in integral form (starting from the work-energy theorem going to Poynting's theorem) to a simple system.
-be able to calculate the bound charge given the electric dipole moment per volume.
-be able to calculate the bound current given the magnetic dipole moment per volume.
-be able to apply perturbative methods to determine the effect of feedback on a system when it is described using a simple block diagram.
-be able to apply Gauss's and Ampere's laws (expressed with D and H) in a simple system to find E and B.

You will be given the triangle diagrams, Poyntings theorem, and the defns related to electric fields in matter.

Review

Ampere's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

In vacuum $\vec{J} = 0$



Find B here

$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{Q}{\text{Area}}; \quad \frac{\partial E}{\partial t} = \frac{1}{\epsilon_0} \frac{I}{\text{Area}}$

$$\int \vec{\nabla} \times \vec{B} \cdot d\vec{a} = \int \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{E} \cdot d\vec{a}$$

$\downarrow$ Stokes

$$\oint \vec{B} \cdot d\vec{r} =$$

Conservation of energy in differential and integral form (see March 24 lecture)

$$W_{net} = \Delta KE$$

$$W_{nc} + W_c = \Delta KE$$

W_c is the work done by the EM fields on the charges.

$$\frac{dW_c}{dt} = -\frac{d}{dt}\underbrace{\int \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) d\tau}_{U_{EM} = \int u_{EM}\, d\tau} - \underbrace{\frac{1}{\mu_0}\oint (\vec{E}\times\vec{B})\cdot d\vec{a}}_{\oint \vec{S}\cdot d\vec{a}}$$ Poyntings theorem

The work done on the charges by the EM force is equal to the decrease in energy stored in the field minus the energy which flowed out of the surface.

$$\frac{dW_c}{dt} = -\frac{dU_{EM}}{dt} - \oint \vec{S}\cdot d\vec{a}$$

If there is no change in KE but just heat generated via a non-conservative force then

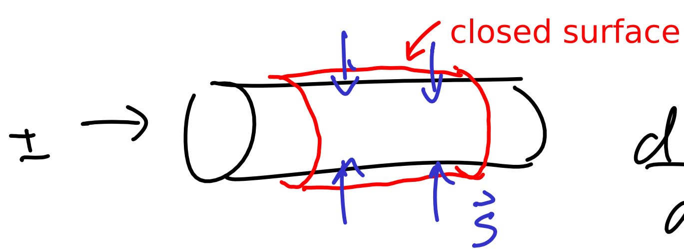
$$W_{net} = \Delta KE$$

$$W_{nc} + W_c = \Delta KE$$

$$\frac{dW_{nc}}{dt} = +\frac{dU_{EM}}{dt} + \oint \vec{S}\cdot d\vec{a}$$

Energy flowing out of surface per time

EM fields don't change within this surface.

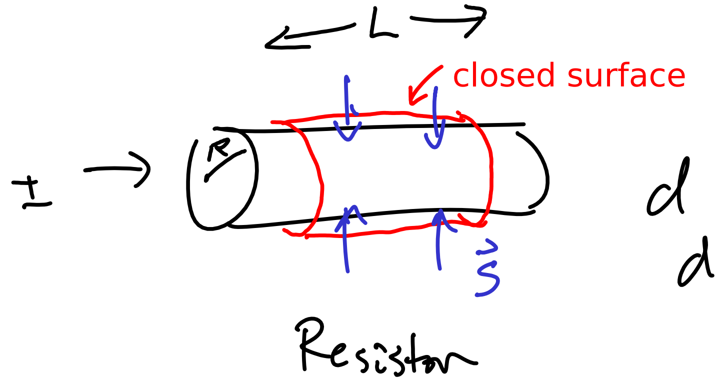


Resistor

$$\frac{dW_{nc}}{dt} = I^2 R = \oint \vec{S}\cdot d\vec{a}$$

If energy is flowing into the surface then this integral is negative and this energy is absorbed.

For the resistor   E = V/L



From Ampere's law

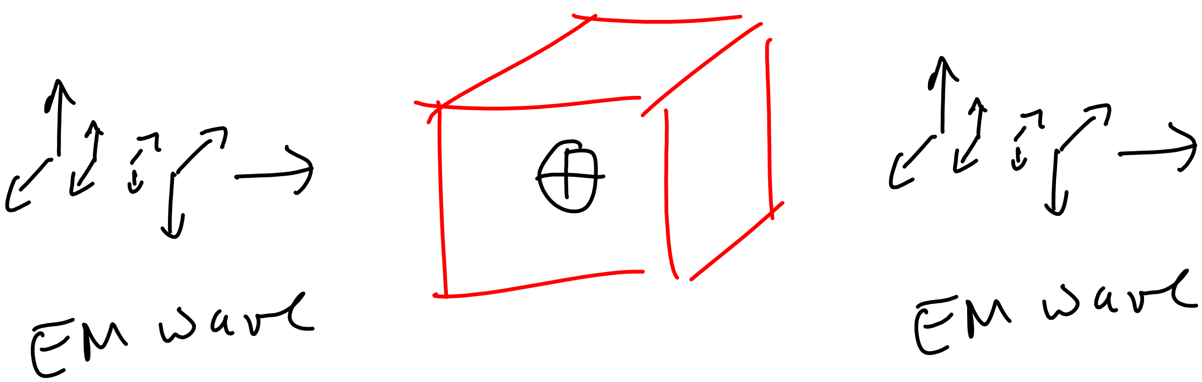
$$B = \frac{\mu_0 I}{2\pi R}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{1}{\mu_0} \frac{V}{L} \frac{\mu_0 I}{2\pi R} = \frac{V I}{2\pi R L}$$

$$\int \vec{S} \cdot d\vec{a} = S \, 2\pi R L = \frac{V I}{2\pi R L} 2\pi R L = V I = I^2 R$$

Example II.

Free charge in empty space. Plane wave is incident from the left.



There is no friction or any other non-conservative force.

$$W_{net} = \Delta KE$$

$$W_{nc} + W_c = \Delta KE$$

$$W_{nc} = 0$$

$$-\frac{dU_{EM}}{dt} - \oint \vec{S}\cdot d\vec{a} = \frac{d}{dt}(\Delta KE)$$

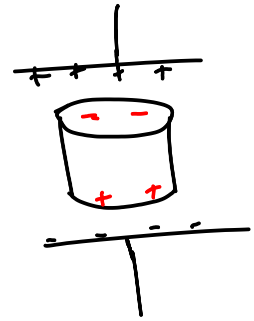
divergence th.

$$-\frac{d}{dt}\int u_{EM}\,d\tau - \int \vec{\nabla}\cdot\vec{S}\,d\tau = \frac{d}{dt}\int u_{KE}\,d\tau$$

$$\frac{\partial}{\partial t}\left(u_{EM} + u_{KE}\right) = -\vec{\nabla}\cdot\vec{S}$$

more

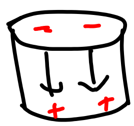
Electric fields in matter



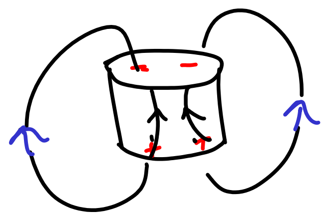
water in a jug
put into a cap
then frozen

Electric field everywhere is calculated by finding the bound charge and free charge then finding the fields from just these charges.

Take the ice out of the cap. Sketch E.



$\vec{P}$



$\vec{E}$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \epsilon \vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon_0(1+\chi_e)}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_f}{\epsilon_0} + \frac{\rho_b}{\epsilon_0}$$

no $\vec{P}$ outside

$\vec{D}$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\vec{\nabla}\cdot\vec{E} + \frac{\vec{\nabla}\cdot\vec{P}}{\epsilon_0} = \frac{\rho_f}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

Questions:

-incongruous: How can this be correct if there can be no electrostatic electric field with a curl? Obviously D has a non-zero curl.

$$\vec{\nabla}\cdot\vec{D} = \rho_{free}$$

$$\vec{\nabla}\times\vec{D} \neq 0$$

Matter in magnetic fields.

Maxwell's eqn $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} = \mu_0(\vec{J}_f + \vec{J}_{bound})$

$\vec{J}_{bound} = \vec{\nabla}\times\vec{M}$

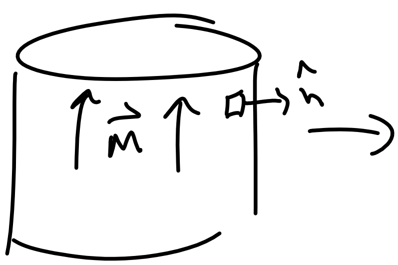
$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \vec{\nabla}\times\vec{M} = \vec{J}_f$$

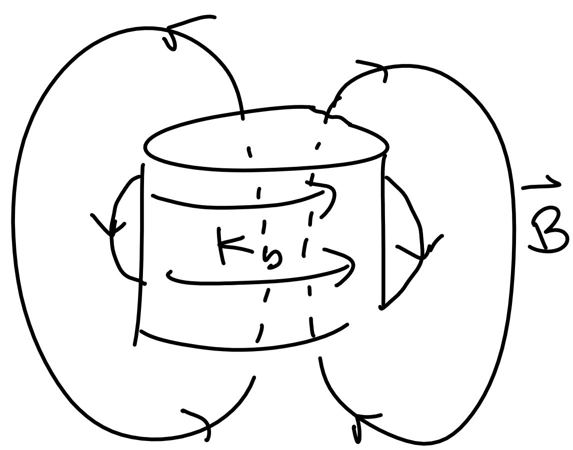
$$\vec{\nabla}\times\Big(\underbrace{\frac{\vec{B}}{\mu_0} - \vec{M}}_{\vec{H}}\Big) = \vec{J}_f$$

Amperes's law in $\vec{H}$

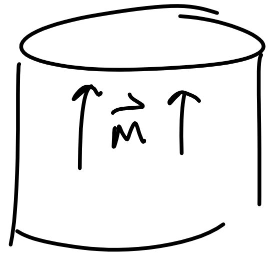
$$\vec{\nabla}\times\vec{H} = \vec{J}_f$$

Linear material $\vec{M} \propto \vec{H}$

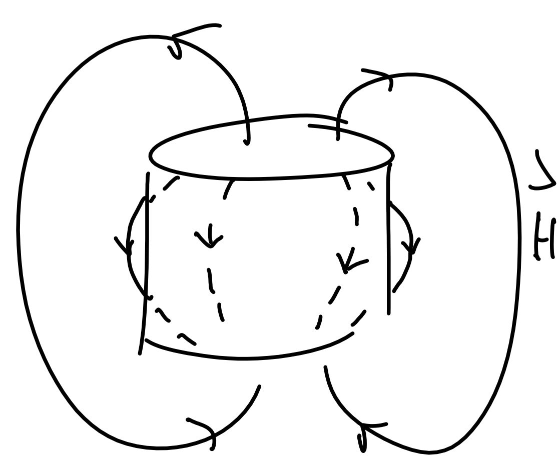




$\vec{K}_b = \vec{M}\times\hat{n}$



Let M be greater than $\frac{B}{\mu_0}$ which is easily satisfied for a ferromagnetic material.



$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

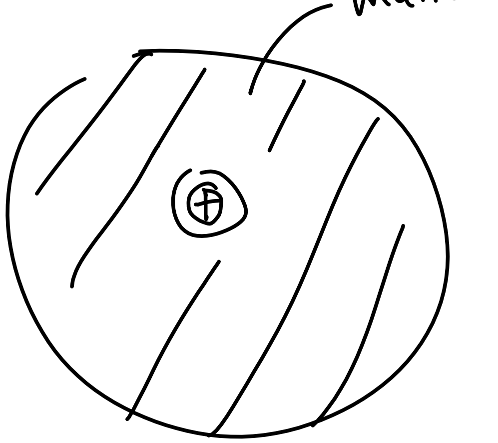
-incongruous: How can this be correct if there can be no magnetic field with non-zero divergence? Obviously H has a non-zero divergence.

$$\vec{\nabla} \times \vec{H} = \vec{J}_f \qquad \vec{\nabla} \cdot \vec{H} \neq 0$$

Look up constitutive relations on wiki.

Examples:

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$



$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\int \vec{\nabla} \cdot \vec{D}\, d\tau = \int \rho_f \, d\tau$$

↓ divergence theorem

$$\oint \vec{D} \cdot d\vec{a} = Q_{f\,\text{enclosed}}$$

$\vec{D} \perp d\vec{a}$ & same for every tile $\Rightarrow \vec{D} \cdot d\vec{a} = |\vec{D}|\, da \cos 0$

$$D \oint da = D 4\pi r^2 = Q_0 \qquad \vec{D} = \frac{Q_0 \hat{r}}{4\pi r^2} = \epsilon \vec{E} \qquad \vec{E} = \frac{Q_0}{4\pi\epsilon r^2}\hat{r}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} \qquad \sigma_b = \vec{P} \cdot \hat{n} \qquad \rho_b = -\vec{\nabla} \cdot \vec{P}$$

$\hat{n} = \hat{r}$

E can also be calculated using only the free and bound charges.

Linear material $\quad \vec{M} = \chi_m \vec{H} \quad$ positive for paramagnetic and negative for diamagnetic stuff

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$$

$$\underbrace{\mu_0(1+\chi_m)}_{\mu} \vec{H} = \vec{B}$$

μ permeability

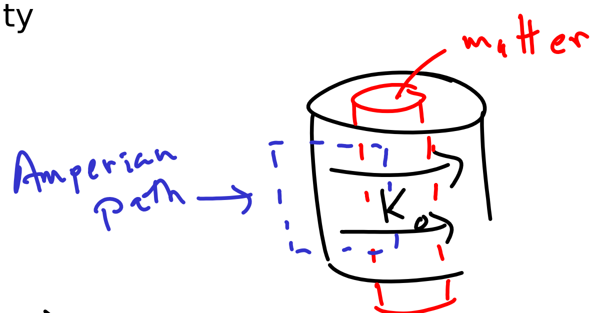
$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\downarrow$$

$$\int \vec{\nabla} \times \vec{H} \cdot d\vec{a} = \int \vec{J}_f \cdot d\vec{a}$$

$\downarrow$ Stokes

$$\oint \vec{H} \cdot d\vec{r} = I_f = K_0 L = \int \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) \cdot d\vec{r}$$



$$\oint \vec{H}\cdot d\vec{r} = I_f = I_{tot} - I_{bound}$$

generated by $\vec{M}$: $K_b = \vec{M}\times\hat{n}$

generates $B_{solenoid}$

generates B_{tot}

Method to find B using Ampere's law, written in terms of H, in a symmetrical current distribution in a linear material:

(1) apply Ampere's law using a path in the region where you want to know B.

(2) solve for H then get B by $\vec{H} = \mu \vec{B}$

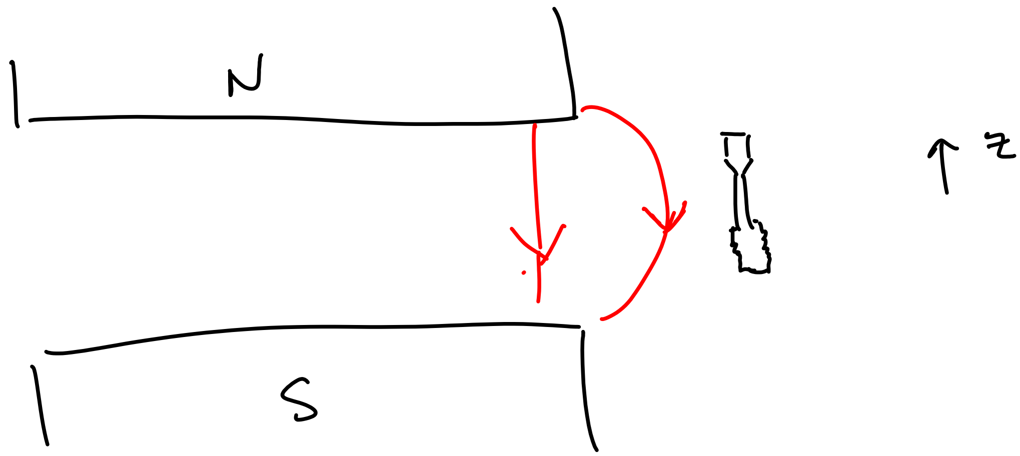
(3) solve for M using H and get the bound currents from

$$\vec{K}_b = \vec{M}\times\hat{n}$$
$$\vec{J}_b = \vec{\nabla}\times\vec{M}$$

Video of screwdriver being sucked into a large magnet.

Questions:

-congruous: How do I calculate that?

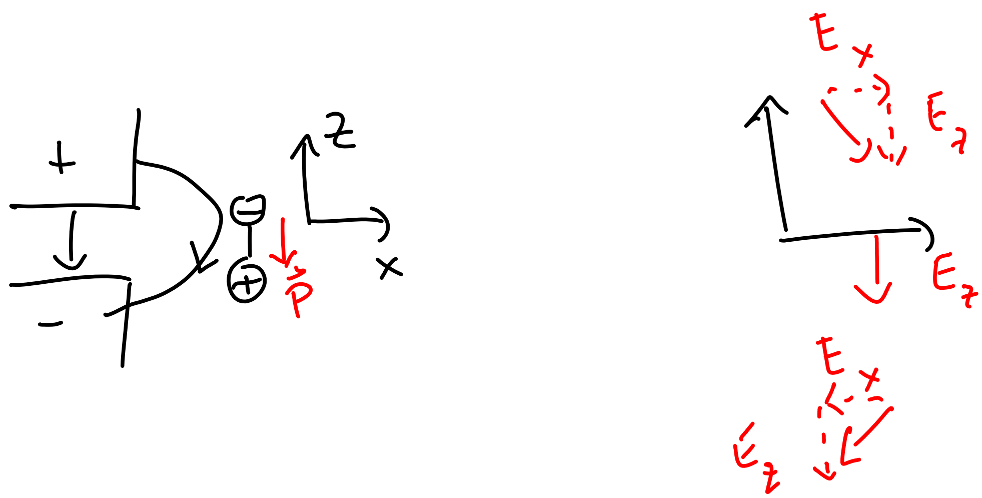


Assume electric dipole in non-uniform electric field since that's the analog we derived a result for.

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$$

$$\vec{p} = -p_0\hat{z}$$

$$\vec{F} = -p_0\frac{\partial}{\partial z}\left(E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E_z(x,y,z)\,\hat{z}\right)$$



E_x changes with z

E_y does not change much with z

$$\vec{F} = -p_0 \frac{\partial}{\partial z} E_x(x,y,z)\,\hat{x} - p_0 \frac{\partial}{\partial z} E_y(x,y,z)\,\hat{y} - p_0 \frac{\partial}{\partial z} E_z(x,y,z)\,\hat{z}$$

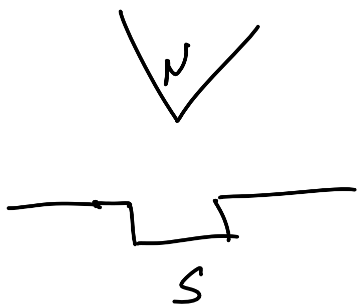
?? ??

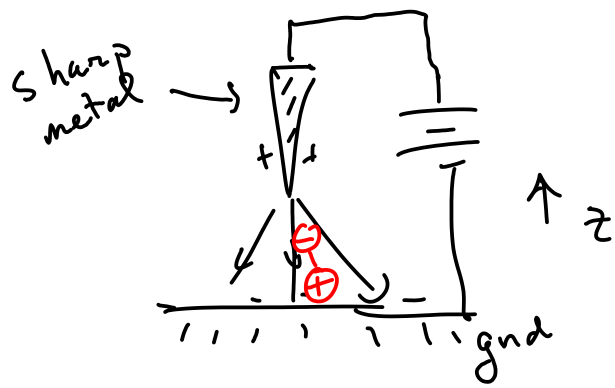
0 0

$$\frac{\Delta E_x}{\Delta z} = \frac{E_{xf} - E_{xi}}{\Delta z} > 0$$

Force is to the left.

Stern-Gerlach





$$\vec{F} = -p_0 \frac{\partial}{\partial z}\left(E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E_z(x,y,z)\,\hat{z} \right)$$

E_z changes with z $\qquad \dfrac{\Delta E_z}{\Delta z} = \dfrac{E_{zf} - E_{zi}}{\Delta z}$

$$E_{zf} = \frac{-7 - (-5)}{1} = -2$$

cancels $-$ to give force up