Phys 361 Homework 6

1) (based on Pollack and Stump 6.4)

a) Show that the potential energy of a dipole of moment $\vec{p}$ sitting in some electric field is given by

$$U = -\vec{p} \cdot \vec{E}$$

b) Now suppose we have an electric dipole sitting in some arbitrary non-uniform E-field, $\vec{E}(\vec{x})$. There will be a force on that dipole, and it'll be related to both the orientation of the dipole and the variation in the E-field. Show that that force is given by:

$$\vec{F} = (\vec{p} \cdot \nabla)\vec{E} = \nabla(\vec{p} \cdot \vec{E})$$

c) Now let's consider the specific case of that same dipole sitting in the field made by a point charge q at the origin. Let the dipole be at point $(0, 0, z_0)$ in Cartesian. Orient it the x-z plane making some angle with the z axis, such that the dipole moment in Cartesian looks like $\vec{p} = p_0(\sin\theta_0\,\hat{\imath} + \cos\theta_0\,\hat{k})$. Show that the force on this dipole from the point charge can be written as:

$$\vec{F}_{on\ dip} = \frac{p_0 q}{4\pi\varepsilon_0 z_0^3}(\sin\theta_0\,\hat{\imath} - 2\cos\theta_0\,\hat{k})$$

Also find the torque on the dipole and the force on the point charge due to the dipole. Make sure that Newton's third law holds.

d) Think physically about what's happening here. A dipole in a uniform electric field will feel zero net force, because the positive and negative charges will feel equal and opposite forces. But what this problem shows us is that a *nonuniform* electric field can create a net force on a dipole (and lots of stuff becomes a dipole when you put it in an electric field and it polarizes). This is one way to start thinking about the technique of optical trapping or optical tweezers. This involves using focused lasers to provide a nonuniform electric field, which then holds a small particle in place. The applications of optical trapping are legion, and some of our own physicists at Mines use it. Note also that the fact that we can reliably trap particles using laser beams suggests that Earnshaw's theorem doesn't hold when we add dynamic fields (like those in electromagnetic radiation).

Check out http://en.wikipedia.org/wiki/Optical_tweezers. Read as much as you like, but especially look at the subsection headed "Physics– General description" and the accompanying diagram. Try to make sense of the phenomenon of optical trapping in the context of the first parts of this problem. Keep it simple – a few sentences and a sketch. Treating optical tweezers formally is more complicated than this, so I just want you to get the gist of it and see that some of this math we do has actual applications.

2) (based on Pollack and Stump 6.9)

Embed a point charge Q in a homogenous, isotropic dielectric medium with dielectric constant κ.

a) If I draw a sphere of radius R centered on charge Q, how much *free* charge will I enclose?

b) How much *bound* charge will that sphere enclose, and how does the result vary with R?

c) Use a sketch and some words to convince me that the kind of variation with R in part (b) makes sense physically.

d) How much *total* charge does that sphere enclose? Don't make this part harder than it is.