PHGN 480: Laser Physics due 4 Sept 2008
Homework 1

1) Suppose we have a two-level quantum system, with an energy difference ΔE separating the levels. The two levels have the same degeneracy. Fill in the following table for the thermal equilibrium population ratio N_2/N_1 for the following combinations of temperatures T and energy differences ΔE.

	T = 100K	T = 300K	T = 1000K
ΔE = 0.0001 eV			
ΔE = 0.05 eV			
ΔE = 3 eV			

The lowest energy difference is characteristic of rotational transitions in molecules, the next corresponds to molecular vibrational transitions, and the highest energy difference is of the order of magnitude of electronic transitions in atoms and molecules.

2) Calculate the electric field strength and the energy density in a plane wave of intensity 100 W/m^2 (=100 mW/cm^2). In SI units, $I = \varepsilon_0 c\, n \left\langle |E|^2 \right\rangle$, where the angular brackets denote a time (cycle average).

3) The intensity of sunlight at the earth's surface is 1 kW/m^2. Calculate the intensity of an image of the sun on the retina assuming:

- The eye's pupil diameter is 2mm (bright-adapted).
- The focal length of the eye is 22.5mm.
- The sun subtends an angle of 0.5^o.

Hint: first calculate the image size by using similar triangles for rays passing from both edges of the sun through the center of the lens (undeviated), then to the retina at one focal length from the lens.

Compare this with the intensity of a 1mW HeNe laser (λ = 632.8nm) at the retina. The beam is Gaussian in profile and enters the eye with a 1/e^2 diameter of 2mm. Hint: the 1/e^2 *radius* of a Gaussian beam focused by a lens of focal length *f* is $w = \dfrac{2}{\pi}\lambda\dfrac{f}{D}$, where *D* is the 1/e^2 *diameter* of the input beam. It is expressed in this way because *f/D* is the effective *F*-number of the focus.

4) Consider the energy level scheme shown in the figure below. Atoms are raised from level 0 to level 2 at a pump rate R_p. The lifetime of levels 1 and 2 are t_1 and t_2 respectively. Assuming that the ground state 0 is not depleted to any significant extent and neglecting stimulated emission:
   a. write the rate equations for the population densities, N_1 and N_2, of level 1 and 2 respectively;
   b. calculate N_1 and N_2 as a function of time
   c. plot the population densities in the fillowing two cases:
      i. τ_1 = 2 μs, τ_2 = 1 μs

ii. $\tau_1 = 1\ \mu s$, $\tau_2 = 2\ \mu s$

Hint: the differential equation for the population of level 1 can be solved by multiplying both sides by the factor exp(t/t1). In this way the left-hand side of the preceding differential equation becomes a perfect differential.