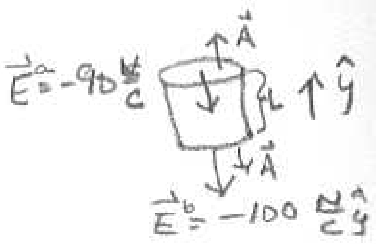
Ch 1 Problems in glass case on 2nd floor.

2.)



$\oint \vec{E}\cdot d\vec{a} = -E^a A + E^b A = \frac{Q_{encl}}{\epsilon_0} = \frac{\int \rho\, d\tau}{\epsilon_0} = \frac{AL\rho}{\epsilon_0}$

$-E^a + E^b = \frac{\rho L}{\epsilon_0}$ or $-(-90) - 100 = \frac{10\rho}{\epsilon_0}$

$\rho = -\frac{10\,\epsilon_0}{10}\ \frac{C}{m^3}$

Problem 2.36

(a) $\boxed{\sigma_a = -\frac{q_a}{4\pi a^2};}$ $\boxed{\sigma_b = -\frac{q_b}{4\pi b^2};}$ $\boxed{\sigma_R = \frac{q_a + q_b}{4\pi R^2}.}$

(b) $\boxed{\mathbf{E}_{\text{out}} = \frac{1}{4\pi\epsilon_0}\frac{q_a + q_b}{r^2}\hat{\mathbf{r}},}$ where $\mathbf{r}$ = vector from center of large sphere.

(c) $\boxed{\mathbf{E}_a = \frac{1}{4\pi\epsilon_0}\frac{q_a}{r_a^2}\hat{\mathbf{r}}_a, \quad \mathbf{E}_b = \frac{1}{4\pi\epsilon_0}\frac{q_b}{r_b^2}\hat{\mathbf{r}}_b,}$ where $\mathbf{r}_a$ ($\mathbf{r}_b$) is the vector from center of cavity a (b).

(d) $\boxed{\text{Zero.}}$

(e) σ_R changes (but not σ_a or σ_b); $\mathbf{E}_{\text{outside}}$ changes (but not $\mathbf{E}_a$ or $\mathbf{E}_b$); force on q_a and q_b still zero.

Problem 2.39

Say the charge on the inner cylinder is Q, for a length L. The field is given by Gauss's law: $\int \mathbf{E}\cdot d\mathbf{a} = E\cdot 2\pi s\cdot L = \frac{1}{\epsilon_0}Q_{\text{enc}} = \frac{1}{\epsilon_0}Q \Rightarrow \mathbf{E} = \frac{Q}{2\pi\epsilon_0 L}\frac{1}{s}\hat{\mathbf{s}}$. Potential difference between the cylinders is

$$V(b) - V(a) = -\int_a^b \mathbf{E}\cdot d\mathbf{l} = -\frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{1}{s}ds = -\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right).$$

As set up here, a is at the higher potential, so $V = V(a) - V(b) = \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$.

$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$, so capacitance *per unit length* is $\boxed{\frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}.}$

Problem 2.40

(a) W = (force)×(distance) = (pressure)×(area)×(distance) = $\boxed{\frac{\epsilon_0}{2}E^2 A\epsilon.}$

(b) W = (energy per unit volume)×(decrease in volume) = $\left(\epsilon_0\frac{E^2}{2}\right)(A\epsilon)$. Same as (a), confirming that the energy lost is equal to the work done.

Problem 2.43

From Prob. 2.12, the field inside a uniformly charged sphere is: $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\mathbf{r}$. So the force per unit volume is $\mathbf{f} = \rho\mathbf{E} = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{Q}{4\pi\epsilon_0 R^3}\right)\mathbf{r} = \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2\mathbf{r}$, and the force in the z direction on $d\tau$ is:

$$dF_z = f_z\,d\tau = \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r\cos\theta(r^2\sin\theta\,dr\,d\theta\,d\phi).$$

The total force on the "northern" hemisphere is:

$$F_z = \int f_z\,d\tau = \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2\int_0^R r^3 dr\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta\int_0^{2\pi}d\phi$$
$$= \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2\left(\frac{R^4}{4}\right)\left(\left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2}\right)(2\pi) = \boxed{\frac{3Q^2}{64\pi\epsilon_0 R^2}.}$$