Saturation effects

2 levels, absorbing transition, constant intensity

$$\frac{dN_2}{dt} = -W(N_2 - N_1) - \frac{N_2}{\tau}, \quad \frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

write in terms of: total $N_t = N_1 + N_2$ $\Big\}$ $N_t = \Delta N + 2N_2$

diff $\Delta N = N_1 - N_2$ $\Big\}$ $\rightarrow N_2 = \frac{1}{2}(N_t - \Delta N)$

$$\frac{d}{dt}(N_1 - N_2) = -2\frac{dN_2}{dt}$$

$$\frac{d}{dt}\Delta N = -2W\Delta N + \frac{2N_2}{\tau} = -2W\Delta N + \frac{N_t}{\tau} - \frac{\Delta N}{\tau}$$

$$= -\Delta N\left(\frac{1}{\tau} + 2W\right) + \frac{1}{\tau}N_t$$

in steady state: $d/dt \rightarrow 0$

$$\Delta N = \frac{N_t}{1 + 2W\tau}$$

$W\tau$ is the key parameter: $W\tau \gg 1$ $\Delta N \rightarrow 0$ or $N_2 = N_1$

Net absorbed power/volume dP/dV

$$\frac{dP}{dV} = h\nu W \Delta N = h\nu \frac{N_t W}{1 + 2W\tau}$$

as $W\tau \gg 1$ $\frac{dP}{dV} \rightarrow h\nu \frac{N_t}{2} \cdot \frac{1}{\tau}$

radiated power: $\frac{N_t}{2}$ lose $h\nu$ in time τ

saturation intensity:

σ = abs. cross-sect.

σI = absorbed energy fluence

$$\frac{\sigma I}{h\nu} = W$$

so $$\frac{\Delta N}{N_t} = \frac{1}{1+2W\tau} = \frac{1}{1+\frac{2\sigma\tau}{h\nu} I}$$

define saturation intensity:

$$I_s \equiv \frac{h\nu}{2\sigma\tau}$$

so $$\frac{\Delta N}{N_t} = \frac{1}{1+I/I_s}$$ in steady-state.

I_s is where the stimulated emission rate = spontaneous em. rate:

absorption: $\sigma = \frac{W}{F} = \frac{W h\nu}{I}$ $\qquad F = I/h\nu$ = photon flux

$$W = \rho B \qquad I = \frac{c\rho}{n}$$

$$= \frac{nI}{c} B$$

skip
work out w/ $I = \frac{c\rho(\nu)\,\Delta\nu}{n}$

$$\rightarrow \sigma = \frac{nI}{c} B \cdot \frac{h\nu}{I} = \frac{n}{c} h\nu B$$

$$I_s = \frac{h\nu}{2} \frac{A}{\frac{n}{c} h\nu B} \qquad \text{w/ } A = 1/\tau$$

$$\frac{n I_s}{c} = \frac{1}{2}\frac{A}{B} \qquad \text{recall that } A = \frac{8\pi h\nu_0^3 n^3}{c^3} B$$

$$\rightarrow I_s = \frac{4\pi h\nu_0^3 n^2}{c^2}$$ for ideal radiatively-broadened line.

Pulsed input $I = I(t)$ duration τ_p

if $\tau_p >> \tau$ (lifetime) and pk $I << I_s$,

$\frac{d\Delta N}{dt} << N_t/\tau$ → similar to CW behavior

$\Delta N(t)$ tracks $I(t)$.

for $\tau_p << \tau$ then we can drop $1/\tau$ terms

$$\frac{d}{dt}\Delta N \doteq -2W\Delta N = -\frac{2\sigma}{h\nu} I(t)\,\Delta N$$

integrate
$$\Delta N(t) = N_t \exp\left[-\frac{2\sigma}{h\nu}\underbrace{\int_0^t I(t)\,dt}_{\text{energy fluence } \Gamma(t)}\right]$$

medium just integrates pumping pulse

define saturation fluence:

$$\Gamma_s = \frac{h\nu}{2\sigma}$$

absorption coeff

$$\alpha = \frac{2\pi^2}{3n\varepsilon_0 ch}|\mu|^2 \Delta N \nu\, g(\nu-\nu_0) \quad \propto \Delta N$$

so since $\Delta N(t) = N_t e^{-\Gamma(t)/\Gamma_s}$

$$\alpha(t) = \alpha_0 e^{-\Gamma(t)/\Gamma_s}$$

Gain saturation

4 - level system

$$\frac{dN_2}{dt} = \underset{\substack{\uparrow \\ \text{pump}}}{R_p} - \underset{\substack{\uparrow \\ \text{stim}}}{WN_2} - \underset{\substack{\uparrow \\ \text{spont}}}{\frac{N_2}{\tau}}$$

steady state

$$\rightarrow N_2 = \frac{R_p \tau}{1 + W\tau}$$

similar behavior to absorption

$$N_2 = \frac{N_{20}}{1 + I/I_s} \qquad I_s = \frac{h\nu}{\sigma\tau}$$

2x higher than for 2-level system

gain coeff saturates

$$g = \frac{g_0}{1 + I/I_s}$$

Working with finite bandwidth sources.

homogeneous, naturally broadened line:

$$\sigma = \frac{2\pi^2}{3n\varepsilon_0 c h} |\mu|^2 \nu \, g(\nu - \nu_0) \qquad 2.4.29$$

$$\text{where } g(\nu-\nu_0) = \frac{2}{\pi \Delta\nu_0} \frac{1}{1 + \left[\frac{2(\nu-\nu_0)}{\Delta\nu_0}\right]^2} \qquad 2.4.8$$

this Lorentzian fcn is normalized

$$\int g(\nu-\nu_0)\, d\nu = 1$$

Absorption rate:

$$W_{12} = \sigma(\nu) F \qquad \text{where } F \text{ is a photon flux}$$

$$= \sigma(\nu) \frac{I}{h\nu} \qquad I = W/cm^2$$

here input is all at one ν

For $I_\nu(\nu)$ spectral intensity, $I_\nu(\nu)\, d\nu = W/cm^2$

Now integrate to get total rate:

$$W_{12} = \int \sigma(\nu) \frac{I_\nu(\nu)}{h\nu} d\nu$$

if BW of input is much larger than $\Delta\nu_0$ of line

$$W_{12} \approx I_\nu(\nu_0) \int \frac{\sigma(\nu)}{h\nu} d\nu$$

$$= I_\nu(\nu_0) \frac{2\pi^2}{3n\varepsilon_0 c h} \frac{|\mu|^2}{h} \int g(\nu-\nu_0)\, d\nu$$

$$W_{12} \rightarrow \frac{2\pi^2}{3n\varepsilon_0 c h^2} |\mu|^2 I_\nu(\nu_0)$$

rewrite in terms of lifetime

$$\tau_{sp}^{-1} = A_{21} = \frac{16\pi^3 \nu_0^3 n |\mu|^2}{3h\varepsilon_0 c^3} \rightarrow \frac{|\mu|^2}{3\varepsilon_0} = \frac{1}{\tau_{sp}} \frac{hc^3}{16\pi^3 \nu_0^3 n}$$

so that for broadband illumination

$$W_{12} = \frac{2\pi^2}{nch^2} \cdot \frac{1}{\tau_{sp}} \frac{hc^3}{16\pi^3 \nu_0^3 n} I_\nu(\nu_0)$$

$$= \frac{c^2}{8\pi n^2 h \nu_0^3} \frac{I_\nu(\nu_0)}{\tau_{sp}}$$

Fluorescence profile

spontaneous emission rate: A_{21} photons/sec.

radiated power $= h\nu \cdot A_{21}$

connect A and B:

$$A_{21} = \underbrace{\frac{8\pi h \nu_0^3 n^3}{c^3}}_{\rho_{vac}(\nu_0)} B_{21}$$

since $\rho(\nu) = \frac{n}{c} I_\nu(\nu)$, effective vac. intensity is

$$I_\nu^{vac}(\nu) = \frac{8\pi h \nu^3}{c^2} n^2$$ this is actually the saturation spectral intensity

radiated power into measured bin $\Delta\nu_m$

$$h\nu W_{21} = h\nu \frac{\sigma(\nu) I_\nu^{vac}(\nu)}{h\nu} \Delta\nu_m$$

$$\propto \nu g(\nu - \nu_0) \nu^3$$

this shifts the fluorescence spectrum compared to actual line.

Gain narrowing of the spectrum

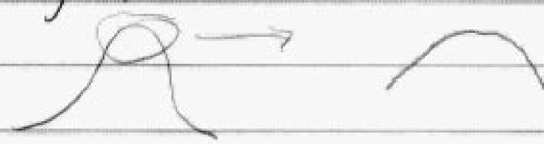
$$\sigma(\nu) \to g(\nu)$$

small signal

$$G_{tot}(\nu) = e^{Ng(\nu)l}$$

$$I_{out}(\nu) = I_{in}(\nu)\, e^{Ng(\nu)l}$$

expand $g(\nu)$ around ν_0 $\to$ $g_0(1-(\Delta\nu/\Delta\nu_0^*)^2)$



$$\to e^{Ng(\nu)l} \doteq e^{Ng_0 l}\, e^{-Ng_0 l(\Delta\nu/\Delta\nu_0^*)^2}$$

new effective line width: $\Delta\nu_0 \to \Delta\nu_0^* / (Ng_0 l)^{1/2}$

$$= \Delta\nu_0^* / \sqrt{\ln(G_{tot})}$$