

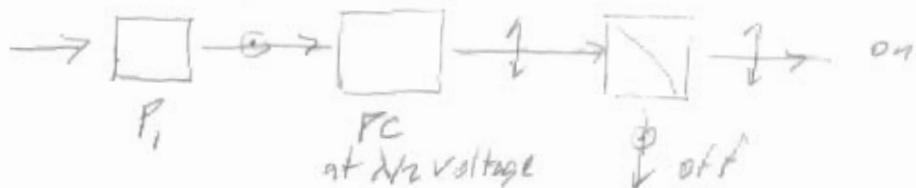
## Electro-optics

Application of a DC (or slowly-varying compared to optical w) field  $\rightarrow$  change in dielectric tensor.

- Pockels: linear in  $\Delta n$  with  $E$ , requires  $\chi^{(2)}$
- Kerr: quadratic in  $E$ , typically  $150 \text{ fm}^2/\text{V}^2$

## Applications:

optical switching - Pockels cell



- pre-polarize light e.g.  $\hat{z} = \hat{y}$
- in off-state input  $\hat{E}$  is along a crystal axis,
- apply voltage: KDP, BBO longitudinal field  
LiNbO<sub>3</sub> transverse

$V$  rotates optical axis  $\rightarrow$  retardation  $\propto V$   
 $V_{\text{max}} \sim 10 \text{ kV}$

i. must switch voltage quickly  $\sim 1-5 \text{ ns}$ .

## E-O modulator

vary voltage on PC  $\rightarrow$  amplitude or phase modulate.

Linear E-O:

$$P_i \sim \chi_{ijk}^{(2)} E_i E_j \sim \chi_{ij}^{(2)} (A_i A_j + \underbrace{A_i^* A_j^* + A_i^* A_j + A_i A_j^*}_{\text{res}}) \quad \begin{matrix} \text{res} \\ \text{W.O.} \\ \text{DC} \end{matrix}$$

process is  $w = w + \theta$

Rather than use  $X_{ik}^{(2)}$ , convention is to modify dielectric tensor. See Boyd for derivation.

Recall index ellipsoid: surface of constant energy density

$$U = \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{E_0}{2} \sum_i E_i \vec{E}_i$$

$$= \frac{1}{2E_0} \left( \frac{D_x^2}{E_{xx}} + \frac{D_y^2}{E_{yy}} + \frac{D_z^2}{E_{zz}} \right) \quad \text{when coordinate system is aligned w/ crystal axes.}$$

we fit

$$X = (2\epsilon_0 u)^{-\frac{1}{2}} D_\alpha \text{ etc. for } Y, Z$$

→ ellipsoid shape:

$$\frac{X^2}{E_{xx}} + \frac{Y^2}{E_{yy}} + \frac{Z^2}{E_{zz}} = 1$$

maxim:  $E_{zz} = n_e^2$   
others  $n_o^2$

in our case here, we will determine the projection of the input fluid onto one of the crystal axes as with wavelets.

$$\text{e.g. } \vec{E}_{in} = \frac{E_0}{\sqrt{2}} (\hat{y} + \hat{z}) \quad \text{axes } X, Y, Z \text{ not aligned with crystal}$$

$$\vec{E}_{out} = \frac{E_0}{\sqrt{2}} (\hat{y} e^{ikx_0} + \hat{z} e^{-ikx_0}) \quad X, Y, Z$$

we will find that applying an E-field changes the crystal axes  
 $\Rightarrow$  uniaxial becomes biaxial

For an arbitrary orientation of the crystal axes, the ellipse can be

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 xy + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 yz = 1$$

$$\begin{array}{ccccccc} \downarrow & & & & & & \\ n_1 & n_{21} & n_{31} & n_{23}-n_{32} & n_{13}-n_{31} & n_{12}-n_{31} \end{array}$$

$$\text{nonlinear response} \quad n_{ij} = n_{ij}^{(0)} + \sum_k r_{ijk} E_k + \sum_l r_{ikl} E_k E_l + \dots$$

so the sum & represent changes to  $\frac{1}{n^2}$

$$\left( \begin{matrix} (\Delta n^2) \\ | \\ 5 \text{ rows} \end{matrix} \right)_1 = \left( \begin{matrix} n_{11} & n_{12} & n_{13} \\ | \\ 5 \text{ rows} \end{matrix} \right) \left( \begin{matrix} E_x \\ E_y \\ E_z \end{matrix} \right)$$

where 1<sup>st</sup> index stands for  
 1  $\rightarrow$  11      4  $\rightarrow$  22 or 22  
 2  $\rightarrow$  22      5  $\rightarrow$  13 or 31  
 3  $\rightarrow$  33      6  $\rightarrow$  12 or 21

↓ applied DC field.

As w/ SHG crystals most r's are zero.

w/ KDP ...  $r_{ijj} = \begin{pmatrix} & \\ & \\ & \\ & \\ & \end{pmatrix}$  KDP  $n_{41} = 8.77 \text{ m/V}$   
 $\bar{n}_{2m}$   $\begin{pmatrix} n_{41} \\ n_{41} \\ n_{63} \end{pmatrix}$   $r_{63} = 10.5 \text{ m/V}$   
 $n_0 = 1.514$

example: no field  $\rightarrow \frac{\Delta^2}{n_0^2} + \frac{\Gamma^2}{n_0^2} + \frac{\Xi^2}{n_e^2} = 1$ ,  $n_e = 1.472$

$$n_{11} = n_{22} = \frac{1}{n_0^2}, \quad n_{33} = \frac{1}{n_e^2}, \quad \text{others} = 0$$

apply field:

$$n_{22} \rightarrow n_{41} E_x$$

$$n_{11,3} \rightarrow n_{41} E_y$$

$$n_{33} \rightarrow r_{63} E_z$$

new ellipsoid is

$$\frac{\Delta^2}{n_0^2} + \frac{\Gamma^2}{n_0^2} + \frac{\Xi^2}{n_e^2} + 2n_{41}E_x\Delta\Xi + 2n_{41}E_y\Delta\Gamma + 2r_{63}E_z\Delta\Gamma = 1$$

apply field along  $Z$  (along optic axis)

- Note: optically there is no difference b/w  $\Delta$  and  $\Xi$  (linear case)

input along  $Z$  ( $E = \frac{1}{2}Z$ ),  $\rightarrow$  no observed birefringence.

turn on voltage, symmetry is broken

$$\rightarrow \text{new axes } X, Y: \quad X = (X - iy)\frac{1}{\sqrt{2}}, \quad Y = (X + iy)\frac{1}{\sqrt{2}}$$

So it is not that the axes are rotated, it appears as an induced birefringence.

Other configurations?

apply field along  $\vec{X}$

$$\rightarrow n_1 X^2 + 2n_3 Z^2 + 2n_4 E_x X Z$$

still would have to propagate along  $\vec{Y}$  to see effect.

Transverse voltage:  $L, NbO_3, GaAs$

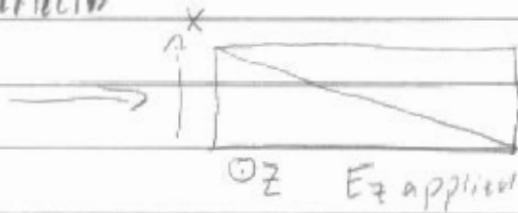
in GaAs  $\Delta\phi \propto VL$   $\text{no more effect w/ larger } L, \text{ small } d$   
 $d$  can use lower voltage.

These  $\rightarrow$  integrated optical devices.

travelling-wave modulators: keep applied voltage moving w/ light



beam deflector



$$\Rightarrow n(x) \propto x \text{ w/ slope } \propto V$$

new ellipsoid:

$$\left( \frac{1}{n_0^2} + r_{63} E_z \right) x^2 + \left( \frac{1}{n_0^2} - r_{63} E_z \right) y^2 + \frac{z^2}{n_0^2} = 1$$

$$\frac{1}{n_x^2} \rightarrow n_x = \left( \frac{1}{n_0^2} + \epsilon \right)^{-\frac{1}{2}} = n_0 (1 + n_0^2 \epsilon)^{-\frac{1}{2}} \approx n_0 - \frac{1}{2} n_0^3 r_{63} E_z$$

$$n_y = n_0 + \frac{1}{2} n_0^3 r_{63} E_z$$

For a modulator or Pockels cell, input is polarized along, say  $\hat{x}$   
 $\rightarrow$  equal projection along  $\hat{x}, \hat{y}$

$$\vec{E}_{\text{out}} = E_0 (\hat{x} e^{ik_0 n_x L} + \hat{y} e^{ik_0 n_y L})$$

$$\Delta\phi = k_0 L (n_y - n_x) = \frac{\omega L}{c} \cdot n_0^3 r_{63} E_z \quad \text{linear in } E_z, \omega$$

$$E_z = V/L \rightarrow \Delta\phi \propto V \quad \text{indep of } L$$

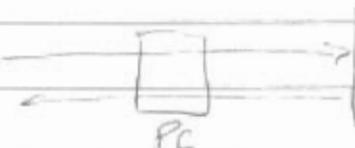
$$\text{at } \Delta\phi = \pi \quad V_{\frac{\pi}{2}} = \frac{\pi c}{\omega n_0^3 r_{63}} \propto \lambda \quad \text{less voltage at short \lambda}$$

$$V_{\frac{\pi}{2}} \sim 10\text{eV for } \lambda \sim 1\mu\text{m}$$

devices:  $\lambda/2$  PC

$\lambda/4$  PC

$\rightarrow \lambda/2$  on 2 passes.



can use as Q-switch, regen, isolation b/w stages.

amp modulator: bias w/  $\lambda/4$   $\rightarrow$  linear regime.

transmission

Phase modulator: input along  $\hat{x}$

$\pi/L$

$\Delta\phi \propto V$

Quasi-phase matching.

- use for non-birefringent materials

consider eqn for  $A_3'$ :

$$A_3' = i \frac{2\omega_3}{n_3 c} d_{eff} A_1 A_2 e^{i(\Delta k \pm k)z}$$

$$\text{let } d_{eff} = d(z) = d_0 \cos Kz$$

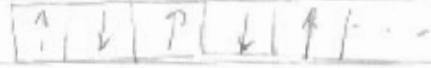
$$A_3' \propto d_0 (e^{iKz} + e^{-iKz}) A_1 A_2 e^{i(\Delta k \pm k)z}$$

$$= d_0 A_1 A_2 \left( e^{i(\Delta k + K)z} + e^{-i(\Delta k - K)z} \right)$$

This modulation allows a buildup in signal if  $\Delta k \pm K = 0$

PPLN: periodically-poled lithium nitrate  $\leftarrow \lambda \rightarrow$

$\chi^{(3)}$  alternates in sign



expand square wave form of  $\chi^{(3)}$  to Fourier series.

$$d(z) = d_{eff} \sum_{m=-\infty}^{\infty} G_m e^{ik_m z} \quad k_m = \frac{2\pi m}{L}$$

can match to diff order 1<sup>st</sup> order QPM is most eff.

Can tailor layering pattern to tailor bandwidth.

Modulation can be in index or intensities.