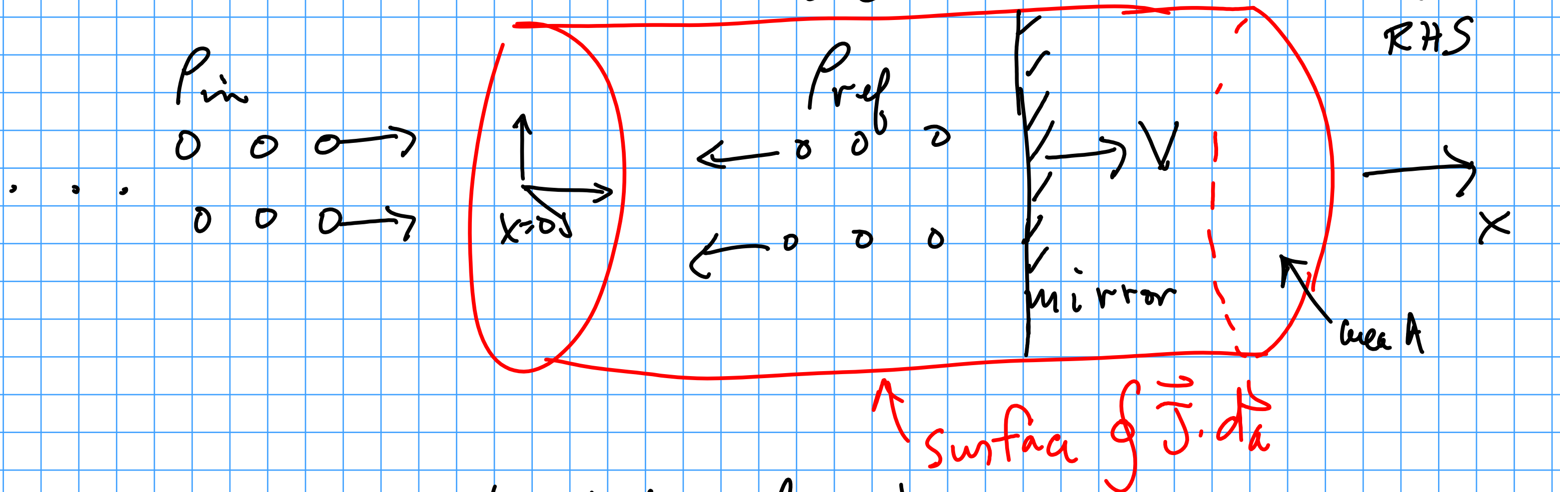
Conservation of charge: $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$ $\quad \vec{J} = \rho\vec{v}$

$$\underbrace{\oint \vec{J}\cdot d\vec{a}}_{LHS} = -\frac{\partial}{\partial t}\int \rho\, d\tau = -\underbrace{\frac{\partial}{\partial t}\,\text{Atoms}_{\text{enclosed}}}_{RHS}$$



$$\rho_{in} = \rho_{ref} = \rho_0 \left(\frac{\text{particles or charges}}{m^3}\right)$$

$$\oint \vec{J}\cdot d\vec{a} = \underbrace{\int \vec{J}\cdot d\vec{a}}_{\text{left end cap}} + \underbrace{\int \cancelto{0}{\vec{J}\cdot d\vec{a}}}_{\text{body}} + \underbrace{\int \overset{0}{\overset{\parallel}{J}}\, da}_{\text{right end cap}}$$

$$LHS = \int_{\text{left end cap}} \left\{\rho_0 v_{in}\hat{x}\cdot(-\hat{x}) + \rho_0 v_{ref}(-\hat{x})\cdot(-\hat{x})\right\} da$$

$v_{in} - 2V$ (ball reflecting from moving wall)

$$= \left\{-\rho_0 v_{in} + \rho_0 \overset{\parallel}{v_{ref}}\right\} A = -2AV\rho_0 = LHS$$

Net flux $\left(m^2 \, \frac{m}{s} \, \frac{\text{atoms}}{m^3} = \frac{\text{atoms}}{s}\right)$ into closed surface

$$RHS = -\frac{\partial\, \text{Atoms enclosed}}{\partial t} = -\frac{\partial}{\partial t}\int \rho\, d\tau = -\frac{\partial}{\partial t}\left\{ \int (\rho_{in} + \rho_{ref})\, d\tau \right\}$$

$$= -\frac{\partial}{\partial t}\left\{ \int_0^{vt} \rho_0 A\, dx + \int_0^{vt} \rho_0 A\, dx \right\} = -2\rho_0 A \frac{\partial}{\partial t}(vt).$$

$$RHS = -2\rho_0 A V = LHS$$

particles inside the volume is increasing at rate $2\rho_0 AV$

$$\left(\frac{\text{atoms}}{m^3}\, m^2\, \frac{m}{s} = \frac{\text{atoms}}{s} \right)$$

particles entering this surfaces (neg number) minus those leaving (positive number) $= -2\rho_0 AV \Rightarrow$

net $\frac{\#}{s}$ entering $= 2\rho_0 AV$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \xrightarrow{1-D} \frac{\partial(\lambda v)}{\partial x} = -\frac{\partial \lambda}{\partial t}$$

Sep variables?

or $$\frac{\partial\left[\lambda(x,t)\, v(x,t)\right]}{\partial x} = -\frac{\partial \lambda(x,t)}{\partial t}$$

PDE must be linear for Sep Variables to work

$$\nabla^2 V = 0 \qquad 1\text{-}D \quad \frac{\partial^2}{\partial x^2} V = 0$$

$V = V_1 + V_2$ & V_1, V_2 solve PDE what about V?

V also must solve PDE

$$\frac{\partial^2}{\partial x}(v_1 + v_2) = \underbrace{\frac{\partial^2 v_1}{\partial x^2}}_{=0} + \underbrace{\frac{\partial^2 v_2}{\partial x}}_{=0} \doteq 0$$

But for cons charge

$$\left.\begin{aligned} \lambda &= \lambda_1 + \lambda_2 \\ v &= v_1 + v_2 \end{aligned}\right\} \text{plug into}$$

Look up "Method Characteristics" on Wikipedia to see how this PDE is solved.

Q

$\vec{J}$

V $\quad$ E

$\vec{A}$ $\quad$ $\vec{B}$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

Should try $\vec{B} = -\vec{\nabla}\varphi$ $\qquad -\vec{\nabla} \times \vec{\nabla}\varphi = \cancel{\mu_0 \vec{J}}\ 0$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x}\varphi & \frac{\partial}{\partial y}\varphi & \frac{\partial}{\partial z}\varphi \end{vmatrix} = 0$$

this doesn't work

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = 0$ always

$$\boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$$

given $\vec{A}$ can find $\vec{B}$

$\vec{\nabla}\times\vec{A} = \vec{B}$

to know $\vec{A}$ we also need $\vec{\nabla}\cdot\vec{A} = ?$

assume $\vec{A} = \vec{A}' + \vec{\nabla}\psi$

scalar function

$$\underbrace{\vec{\nabla}\times\vec{A}}_{B} = \vec{\nabla}\times(A' + \vec{\nabla}\psi) = \underbrace{\vec{\nabla}\times\vec{A}'}_{B} + \underbrace{\vec{\nabla}\times\vec{\nabla}\psi}_{0}$$

Gauge

$$\vec{\nabla}\cdot\vec{A} = \vec{\nabla}\cdot\vec{A}' + \underbrace{\vec{\nabla}\cdot\vec{\nabla}}\psi = \vec{\nabla}\cdot\vec{A}' + \underbrace{\nabla^2\psi}_{\text{anything}} = \phi$$

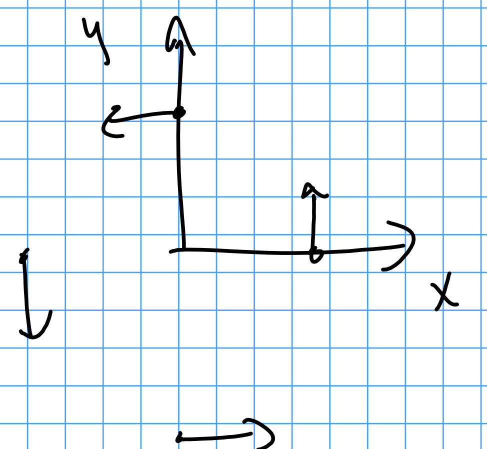
$$\boxed{\begin{aligned}\vec{\nabla}\times\vec{A} &= \vec{B}\\ \vec{\nabla}\cdot\vec{A} &= 0\end{aligned}}$$

Lorentz Gauge

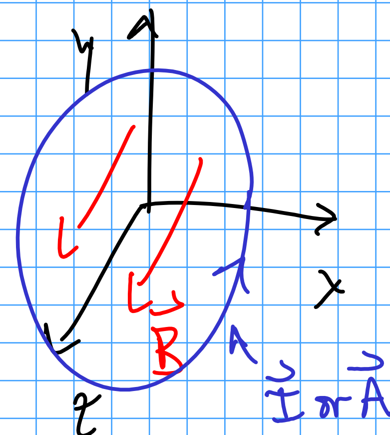
Ex:

$$\vec{A} = -\frac{B_0 y}{2}\hat{x} + \frac{B_0 x}{2}\hat{y} + 0\hat{z}$$

$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{B_0 y}{2} & \frac{B_0 x}{2} & 0 \end{vmatrix} = B_0 \hat{k}$$



$\vec{A}$



Note that $\vec{A} = B_0 y \hat{x}$ gives the same $\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0 y & 0 & 0 \end{vmatrix} =$

$= \hat{z} B_0$