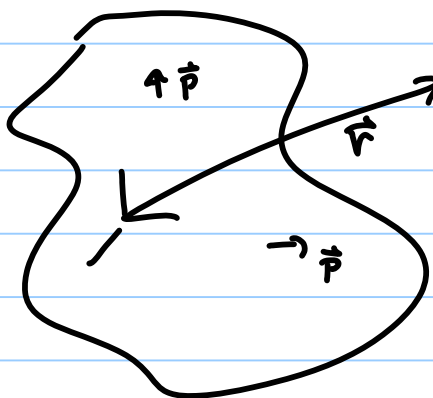


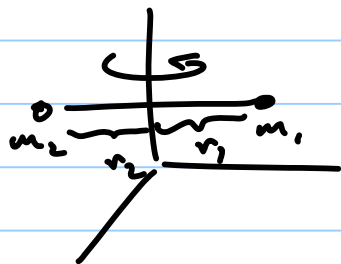
$$V = \frac{1}{4\pi\epsilon_0} \frac{q_{tot}}{r}$$

$$V_{dipole} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

$$\vec{E}_{dipole} = -\vec{\nabla} V$$



Moment of inertia



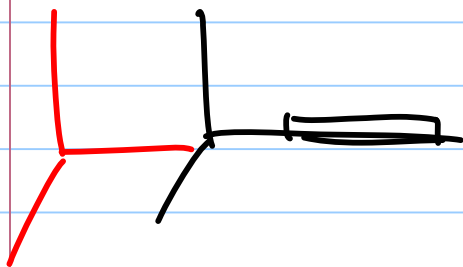
$$I = \sum_i m_i r_i^2 \rightarrow \int r^2 dm$$

depends on location of axis

$\rho d\tau$

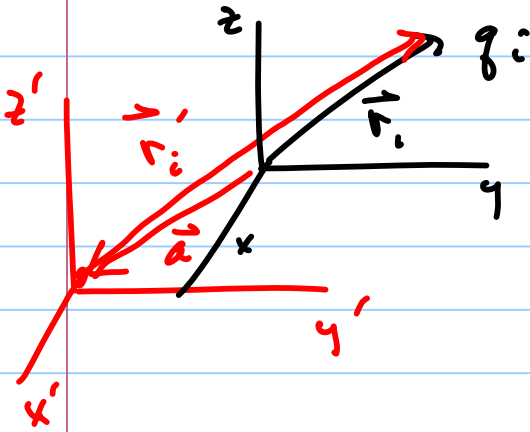
Center of mass

$$M_{tot} X_{cm} = \sum_i m_i x_i \rightarrow \int x dm$$



Dipole mom

$$\vec{p} = \sum_i q_i \vec{r}_i$$



$$\vec{a} + \vec{r}_i' = \vec{r}_i$$

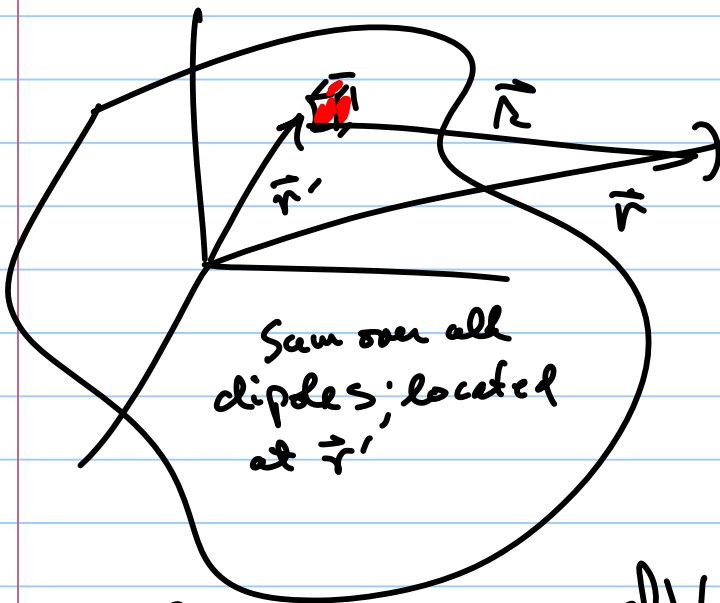
$$\vec{p}' = \sum_i q_i \vec{r}_i' = \sum_i q_i (\vec{r}_i - \vec{a})$$

$$= \underbrace{\sum_i q_i \vec{r}_i}_{\vec{p}} - \underbrace{\sum_i q_i \vec{a}}_{\vec{a} \sum_i q_i}$$

Q_{tot}

if dipole has $Q_{tot} = 0$

$\vec{p}' = \vec{p}$ indep of coord location

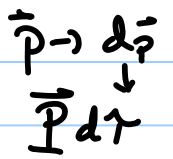
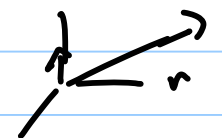


permanent dipoles are embedded in material

find $V(\vec{r})$

$$V_{dipole} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

$$dV_{dipole} = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot \hat{r} d\vec{r}'}{r^2}$$



dipole mom/ vol

$$\sum_i m_i r_i^2 \rightarrow \int r^2 dm$$

$\vec{p} \cdot \vec{r}$ change vol

\vec{P}

dipole mom
vol

$\sim \rho$ charge (mean)
vol

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\vec{P}(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\tau' = \frac{1}{4\pi\epsilon_0} \vec{P} \cdot \vec{\nabla} \frac{1}{|\vec{r} - \vec{r}'|} d\tau'$$

$$\vec{\nabla} \frac{1}{|\vec{r} - \vec{r}'|} = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{A} \cdot \vec{\nabla} f = \vec{\nabla} \cdot f \vec{A} - f \vec{\nabla} \cdot \vec{A}$$

$$\vec{\nabla} \frac{1}{\sqrt{(x-x')^2 + \dots}} = \hat{x}' \frac{\partial}{\partial x'} \frac{1}{\sqrt{\dots}} + \dots$$

$$V = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \frac{\vec{P}}{|\vec{r} - \vec{r}'|} d\tau' = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{\nabla} \cdot \vec{P}}{|\vec{r} - \vec{r}'|} d\tau'$$

$$V = \oint \frac{\vec{P}}{|\vec{r} - \vec{r}'|} \cdot d\vec{a} + \frac{1}{4\pi\epsilon_0} \int \frac{-\vec{\nabla} \cdot \vec{P}}{|\vec{r} - \vec{r}'|} d\tau'$$

$$\int \vec{\nabla} \cdot \vec{E} d\tau = \oint \vec{E} \cdot d\vec{a}$$

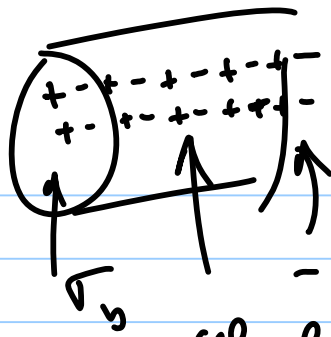
$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho d\tau'}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b da'}{|\vec{r} - \vec{r}'|} + \frac{1}{4\pi\epsilon_0} \int \frac{\rho_b d\tau'}{|\vec{r} - \vec{r}'|}$$

$$\sigma_b = \vec{P} \cdot \hat{n} \text{ on surface}$$

$$d\vec{a} = \hat{n} da$$

$$\rho_b = -\vec{\nabla}' \cdot \vec{P}(\vec{r}')$$



have only σ

all change cancels