Lecture 40

Learning objectives for the final

The learning objectives for exam 1 are that you will be able to:
-find E given the charge distribution or find the charge distribution give E using Gauss's law in a symmetric geometry.
-derive (explain the steps in) the previous result.
-memorize the divergence theorem
-be able to derive the integral form of conservation of charge given the differential form and be able to explain what it means.
-derive the area and volume of a sphere or cylinder using calculus.
-apply the divergence theorem for a simple vector function.

The learning objectives for exam 2 are that you will be able to:
-find B given a symmetric current distribution using Ampere's law.
-derive (explain the steps in) the previous result.
-memorize Stokes theorem.
-apply Stoke's theorem for a simple vector function.
-set up an integral to determine the force on a moving charge distribution in steady state (charge moving along a line, surface, or volume).

Formulas: Memorize Stokes and the divergence theorems. Also memorize Gauss's and Ampere's laws.

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The learning objectives for exam 3 are that you will be able to:
- memorize Stokes theorem
- memorize Divergence theorem
- memorize differential and integral forms of conservation of charge
- understand Ohm's law and why the relaxation method can be used to determine the current density.
- know how to calculate the vector potential given a current distribution
- understand how to calculate the work required to assemble a current distribution in the many ways we discussed.
- be able to calculate the Faraday E due to a changing B using the integral form of Faraday's law.

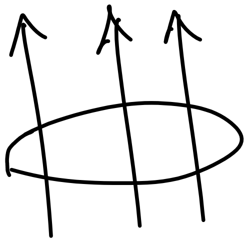
Learning objectives for exam 4 are that you will be able to:

- be able to calculate the magnetic field due to a time changing electric field.
- be able to apply conservation of energy in integral form (starting from the work-energy theorem going to Poynting's theorem) to a simple system.
- be able to calculate the bound charge given the electric dipole moment per volume.
- be able to calculate the bound current given the magnetic dipole moment per volume.
- be able to apply perturbative methods to determine the effect of feedback on a system when it is described using a simple block diagram.
- be able to apply Gauss's and Ampere's laws (expressed with D and H) in a simple system to find E and B.

Formulas: Memorize Faraday's law and Ampere's law with the displacement current.

Questions about flux compression?

incongruous: How can energy and momentum be conserved?



$I_0 = B_0 A_0$

$\Phi_f = B_f A_f$

Review of metal detector:
(1) simplify the problem associated with you idea.
(2) calculate the effect to see if the idea will work. This is part of the step
Develop a decision matrix or rubric.

-congruous: How do I calculate the effect of a copper wire loop which is located 30 cm away from a wire loop inductor (the sensor)?

Review of EM force projectile.

Group project for participation credit: Design an EM device which shoots a projectile.

Motivation:
-collide two materials to generate nuclear fusion
-launch material into orbit
-new type of gun

Look up the "engineering design process."

Define the problem: Project matter using EM fields and forces.

Background reserach:

Define requirements: Move matter faster than with chemical (gunpowder) or mechanical (bow and arrow) means.

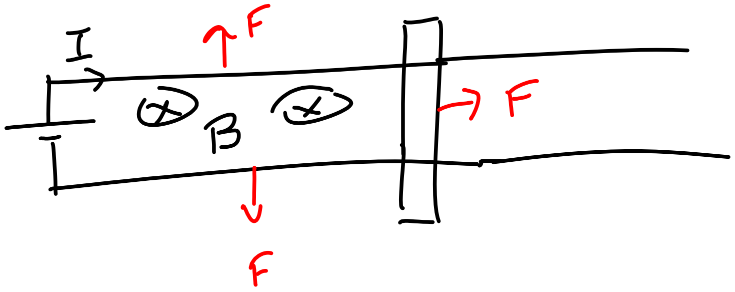
Generate ideas: (put them in a simple form to be able to calculate the effect)

Generate ideas is unrestrained. Later in developing the decision matric do some positive and negative critical thinking about these ideas.

-place a large charge on the projectile and a like charge on the "gun."
Use q E as the force.

-discharge a capacitor through a conducting wire. The qvB force will distort the wire into a circle. If the force is large enough the wire will explode.

Simple version



Questions: **Develop a decision matrix or rubric.**

-congruous: How to I calculate the force on the object and its speed?

$$\vec{F} = I\, d\vec{r} \times \vec{B} \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

-congruous: How do I calculate the current?

$$Emf - IR - Emf_{back} = 0$$

Faraday's law (same issue in flux compression)

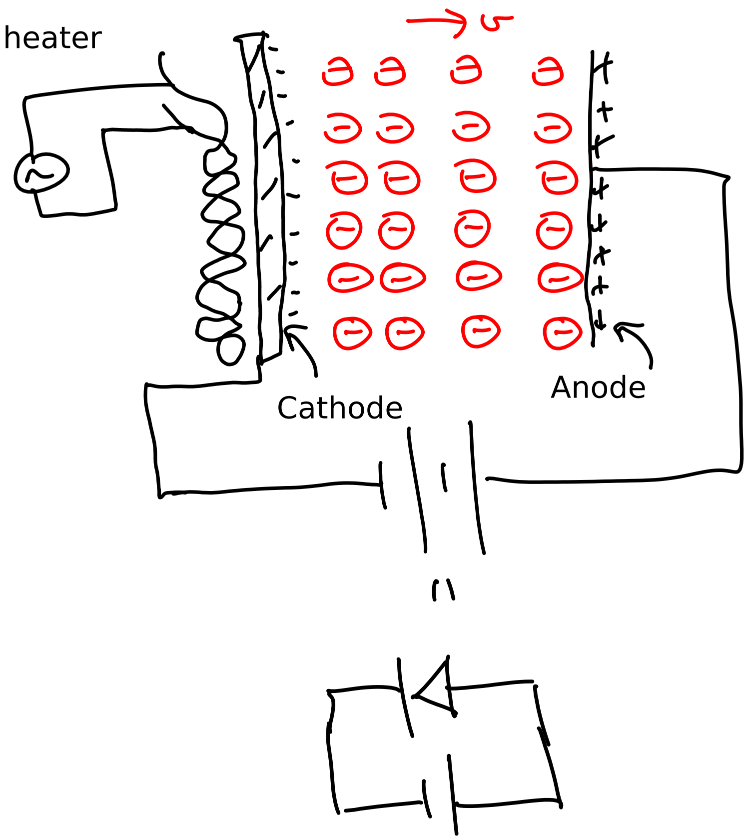
$$Emf_{back} = -\frac{d\Phi_B}{dt} \qquad \Phi \equiv LI$$

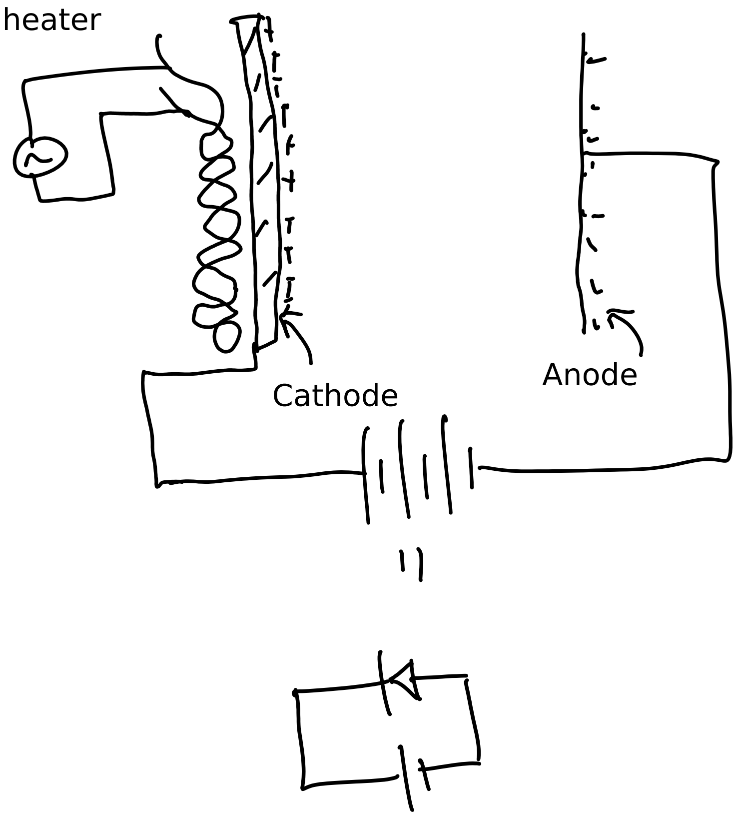
$$\frac{d\Phi_B}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt}$$

-congruous: How do I calculate the energy that goes to the projectile?

-congruous: How is momentum conserved? How much does the gun recoil?

Group project: Vacuum diode (electron gun) in steady state





What are the fundamental equations which are needed to determine the current in this diode?

Simplifying assumptions: all motion is in 1-D

Field eqn: E or V

① $$\frac{d^2V}{dx^2} = -\frac{\rho(x)}{\epsilon_0} \qquad V(x)$$

Dynamics (motion of charges)

② $$\frac{1}{2}mv^2 = qV(x)$$

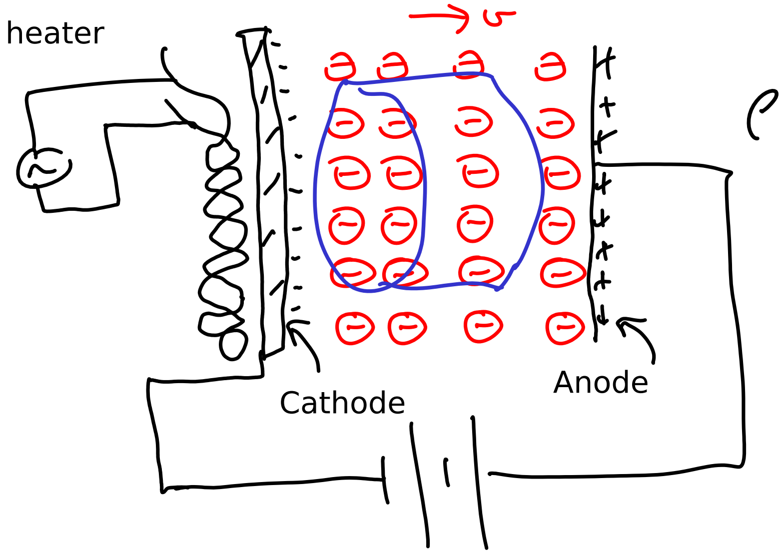
Boundary conditions

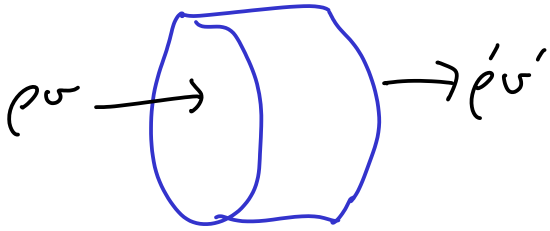
③ $$V(x=0) = 0 \qquad V(x=D) = V_0$$

Conservation of charge

④ $$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} = 0 \xrightarrow[\text{th}]{\text{divergence}} \oint \vec{J}\cdot d\vec{a} = 0$$

$$\vec{J} = \rho v\hat{x} \qquad JA = I$$





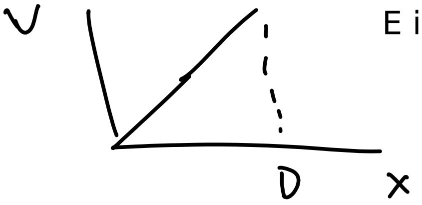
$$\rho v A = \rho' v' A$$

④ $\boxed{\rho(x)\, v(x) = \text{constant}}$

Simplify the problem before addressing its complexity by solving with no space charge (one electron moving).

$$V = E_0 x$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{x} = -E_0\hat{x}$$



E is constant

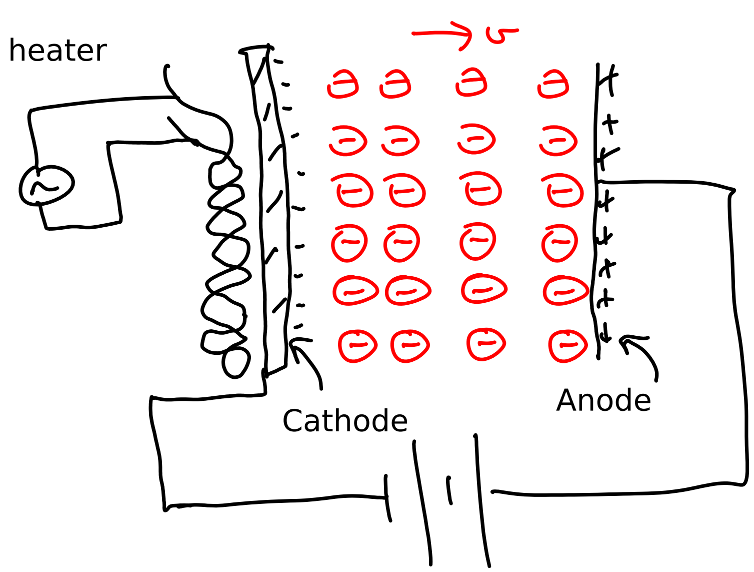
$$\frac{1}{2}mv^2 = q\,V(x) = qE_0 x$$

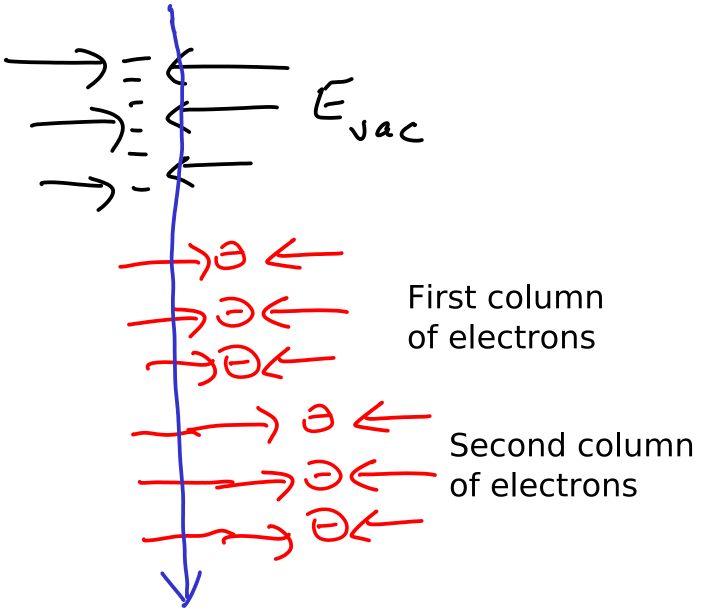
How will the problem with space charge differ?

$$J = \text{constant} = \underset{\text{large on left}}{\rho}\,\overset{\text{small on left}}{v}\Big|_{\text{left}} = \underset{\text{small}}{\rho}\,\overset{\text{large}}{v}\Big|_{\text{right}}$$

How would you expect E to look?

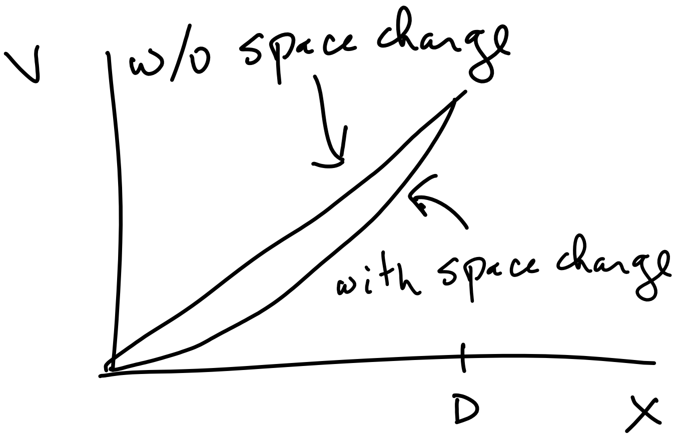
Dense charges on the left shield the anode from electrons to the right.

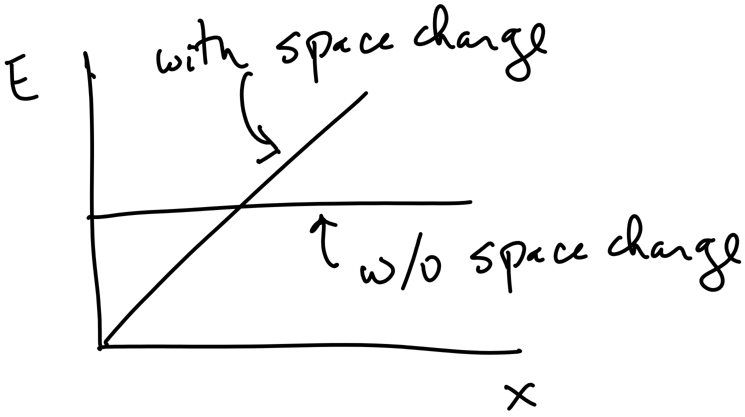




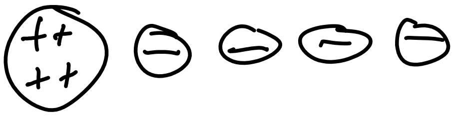
These E's add to reduce the E in vacuum. This is shielding.

How would you expect V to look from this shielding effect?





Such screening occurs in atoms with more than 1 electron. It is screening of the nuclear charge by electrons closer to the nucleous.



Mathematical soln:

We want solns to Poisson's eqn which meet bndry conditions.

Lots of solns but need V and dV/dx.

Use physical argument to impose another condition: For a given V0 the repulsive effect fo space charge is so greate that no more electrons could get across even if more were to be released from the cathode. This forces E = 0 at cathode.

$$\frac{d^2V}{dx^2} = -\frac{\rho(x)}{\epsilon_0} = -\frac{1}{\epsilon_0}\frac{I}{Av(x)} \quad \text{using } \rho v = \frac{I}{A}$$

← area of cap

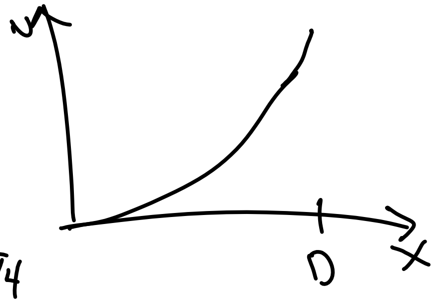
$$\frac{d^2V}{dx^2} = -\frac{1}{\epsilon_0}\frac{I}{Av(x)} = -\frac{I}{\epsilon_0 A}\sqrt{\frac{m}{2qV(x)}} \quad \text{using } \frac{1}{2}mv^2 = qV(x)$$

Use bndry conditions: $V(0) = \left.\frac{dV}{dx}\right|_{x=0} = 0$

Mathematica

$$V(x) = V_0\left(\frac{x}{D}\right)^{4/3}$$

$$V_0 = \left(\frac{81 I^2 m}{32 \epsilon_0^2 A^2 q}\right)^{1/3} \frac{1}{D^{3/4}}$$



This is one of a few examples which have closed form solutions. For quantum look up "list of quantum mechanical systems with analytic solutions"

You don't need to know everything about the subject before you can contribute to new knowledge. Being aware of this encourages you to think of novel ideas rather than waiting until you know it all (you'll never be there).

Example: the axe which has been around for thousands of years.
Look up "Physics-exploiting axe splits wood in record time."