

Physics 200: Fundamental Equations

Maxwell's Equations

Gauss's Law for Electric Fields: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = 4\pi k Q_{encl}$

Gauss's Law for Magnetic Fields: $\oint \vec{B} \cdot d\vec{A} = 0$

Electric Flux: $\Phi_E = \int \vec{E} \cdot d\vec{A}$; Magnetic Flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Ampère/Maxwell: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

Fields, Forces and Energy

Electric Field: $d\vec{E} = \frac{kQ}{r^2} \hat{r} = \frac{kQ}{r^3} \vec{r}$; $\vec{F}_E = q\vec{E}$ at q

Electric Potential (Voltage): $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$; $E_x = -\frac{dV}{dx}$; $dV = \frac{kQ}{r}$

Electrostatic Energy: $U_{of q} = qV$

Dielectrics: $\epsilon = \kappa \epsilon_0$

Magnetic Field: $d\vec{B} = \frac{\mu_0 I dl \sin \theta}{4\pi r^2} = \frac{\mu_0 I dl \times \vec{r}}{4\pi r^3}$

Magnetic Force: $d\vec{F} = I dl \times \vec{B}$; $\vec{F}_B = q\vec{v} \times \vec{B}$

Magnetic Dipole: $\vec{\mu} = NIA\hat{n}$; $\vec{\tau} = \vec{\mu} \times \vec{B}$

Circuits

Resistors: $dR = \frac{\rho dl}{A}$; $R_{series} = \sum_i R_i$; $R_{parallel} = (\sum_i R_i^{-1})^{-1}$

Capacitors: $C = \frac{Q}{V}$; $U_C = \frac{1}{2} CV^2$; $C_{series} = (\sum_i C_i^{-1})^{-1}$; $C_{parallel} = \sum_i C_i$; $C = \kappa \epsilon_0$

Ohm's Law: $V = IR$

Current: $I = \frac{dQ}{dt} = n|q|v_d A$

Power: $P = IV$

Kirchoff's Laws: $\sum_{loop} V_i = 0$; $\sum I_{in} = \sum I_{out}$

RC & LR Circuits: Charging and Discharging equations take the form of $e^{-t/\tau}$ and $1 - e^{-t/\tau}$

$\tau_{RC} = RC$; $\tau_{LR} = \frac{L}{R}$

AC Circuits: $X_C = \frac{1}{\omega C}$; $V_C = IX_C$; $Z = \sqrt{R^2 + X_C^2}$; $V = IZ$; $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$

Inductors: $\vec{v}_{ind} = -\frac{d\vec{A}}{dt}$; $L = \frac{N^2 \mu_0 \mu_r}{l}$; $U_L = \frac{1}{2} LI^2$

Inductance: $M_{12} = \frac{\mu_0 N_1 N_2}{4\pi r}$; $\vec{M} = -\frac{\partial U}{\partial \vec{M}}$

Electromagnetic Waves, Optics and Field Energy Density

Field Energy Density: $u_{EM} = \frac{1}{2} \epsilon_0 E^2$; $u_{EM} = \frac{1}{2\mu_0} B^2$

Momentum: $\vec{p} = \frac{U}{c} \hat{n}$

Wave Properties: $v = \lambda f$; $\vec{k} = \frac{2\pi}{\lambda} \hat{n}$; $\omega = 2\pi f$; $\vec{E} = \frac{c}{\omega} \vec{k} \times \vec{B}$

Intensity: $I = \langle \vec{E} \times \vec{H} \rangle = \frac{1}{2} c \epsilon_0 E_0^2$

Reflection/Refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$; $n_1 \cos \theta_1 = n_2 \cos \theta_2$; $n_2 \sin \theta_2 = n_1 \sin \theta_1$

Additional Information/Useful Constants

Common Electric Fields: $E_{inf sheet} = \frac{\sigma}{2\epsilon_0}$; $E_{inf line} = \frac{2k\lambda}{r}$; $E_{charged ring} = \frac{kQx}{(x^2+a^2)^{3/2}}$; $C_{parallel plate} = \frac{\epsilon_0 A}{d}$

Common Magnetic Fields: $B_{inf wire} = \frac{\mu_0 I}{2\pi r}$; $B_{solenoid} = \mu_0 n I$; $L_{solenoid} = \mu_0 n^2 A l$; $B_{current loop} = \frac{\mu_0 N I R^2}{2(x^2+R^2)^{3/2}}$

Fundamental Charge: $e = 1.602 \times 10^{-19} C$; Electron Mass: $m_e = 9.109 \times 10^{-31} kg$

Proton Mass: $m_p = 1.673 \times 10^{-27} kg$

$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$; $\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$

$\mu_0 = 4\pi \times 10^{-7} \approx 12.566 \times 10^{-7} \frac{Tm}{A}$

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \frac{m}{s}$

Vector Derivatives

Cartesian Coordinates

$$d\ell = \hat{i} dx + \hat{j} dy + \hat{k} dz, \quad dV = dx dy dz$$

$$\nabla f = \hat{i} \frac{\partial f}{\partial x} + \hat{j} \frac{\partial f}{\partial y} + \hat{k} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates

$$d\ell = \hat{r} dr + \hat{\phi} r d\phi + \hat{k} dz, \quad dV = r dr d\phi dz$$

$$\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial f}{\partial \phi} + \hat{k} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{r} \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{k} \left[\frac{1}{r} \frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$d\ell = \hat{r} dr + \hat{\theta} r d\theta + \hat{\phi} r \sin \theta d\phi, \quad dV = r^2 \sin \theta dr d\theta d\phi$$

$$\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{\hat{r}}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] + \frac{\hat{\theta}}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] + \frac{\hat{\phi}}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

Vector Formulas

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = (\mathbf{C} \times \mathbf{A}) \cdot \mathbf{B} = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

Derivatives of Sums

$$\nabla(f + g) = \nabla f + \nabla g$$

$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$$

$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$$

Derivatives of Products

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$$

Second Derivatives

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla f) = 0$$

Integral Theorems

$$\int_V (\nabla \cdot \mathbf{A}) dV = \oint_S \mathbf{A} \cdot \hat{\mathbf{n}} dS \quad \text{Gauss's (divergence) Theorem}$$

$$\int_S (\nabla \times \mathbf{A}) \cdot \hat{\mathbf{n}} dS = \oint_C \mathbf{A} \cdot d\mathbf{l} \quad \text{Stokes's (curl) Theorem}$$

$$\int_a^b (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$$

$$\int_V (f\nabla^2 g - g\nabla^2 f) dV = \oint_S (f\nabla g - g\nabla f) \cdot \hat{\mathbf{n}} dS \quad \text{Green's Theorem}$$

Chapter 5, 6, 7, 8 useful relationships

Separation of variables general solutions

Cartesian: $V(x, y, z)$ as combinations of $\cos(kx) + \sin(kx)$ or $\cosh(kx) + \sinh(kx)$
or $e^{kx} + e^{-kx}$

Spherical: $V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-l-1}) P_l(\cos(\theta))$

Cylindrical: $V(r, \varphi) = A \ln(r) + B + \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-n}) (C_n \cos(n\varphi) + D_n \sin(n\varphi))$

Legendre polynomials: $P_0(x) = 1$; $P_1(x) = x$; $P_2(x) = \frac{3}{2}x^2 - 1$; $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$

Legendre orthogonality relationship: $\int_{-1}^1 P_n(x) P_l(x) dx = \delta_{nl} \frac{2}{2n+1}$

Polarization relationships

$$\vec{p} = \alpha \vec{E} \quad \vec{P} = \chi_e \varepsilon_0 \vec{E} \quad \varepsilon = \varepsilon_0 (1 + \chi_e) \quad \kappa = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$$

$$\sigma_b = \hat{n} \cdot \vec{P} \quad \rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$\vec{P} = n \vec{p} \quad \alpha = \frac{3\varepsilon_0 \kappa - 1}{n \kappa + 2}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E} \quad \vec{\nabla} \cdot \vec{D} = \rho_f$$

Current and current densities

$$I = \frac{dQ}{dt} = q n_L v \quad dI = \vec{j} \cdot d\vec{A} \quad \vec{j} = q n \vec{v} \quad \vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

$$\vec{j} = \sigma \vec{E} \quad dI = \vec{K} \cdot \hat{e}_{\perp} dl \quad RC = \frac{\varepsilon}{\sigma}$$

Magnetic vector potential

$$\vec{B} = \nabla \times \vec{A} \quad \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$$

Chapter 9, 10, 11 useful relationships

Magnetization relationships

$$\vec{M} = \chi_m \vec{H} \quad \mu = \mu_0(1 + \chi_m)$$

$$\vec{J}_b = \nabla \times \vec{M} \quad \vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

Forms of Faraday's Law

$$EMF = -\frac{d\Phi}{dt} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

Momentum & energy

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \quad u_{em} = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \quad \frac{d\vec{P}}{dV} = \mu_0 \epsilon_0 \vec{S}$$

Boundary conditions in matter

$$D_{2,perp} - D_{1,perp} = \sigma_f \quad B_{1,perp} = B_{2,perp}$$

$$\vec{E}_{1,parallel} = \vec{E}_{2,parallel} \quad \vec{H}_{2,parallel} - \vec{H}_{1,parallel} = \vec{K}_f \times \hat{n}$$

Potentials & Gauges

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad \vec{B} = \nabla \times \vec{A} \quad \text{Coulomb:} \quad \nabla \cdot \vec{A} = 0$$

$$\text{Transforms:} \quad V \rightarrow V - \frac{\partial f}{\partial t} \quad \vec{A} \rightarrow \vec{A} + \nabla f \quad \text{Lorentz:} \quad \nabla \cdot \vec{A} = -\epsilon_0 \mu_0 \frac{\partial V}{\partial t}$$