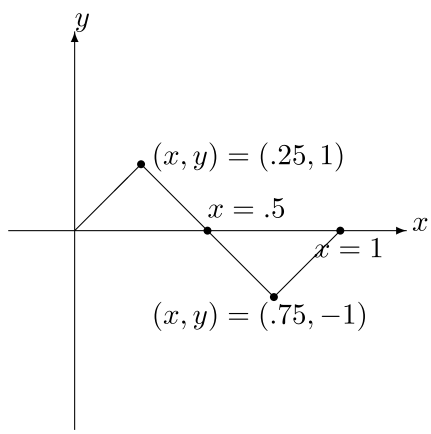
MATH348 - April 20, 2012 **NAME:**
Makeup Exam II - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

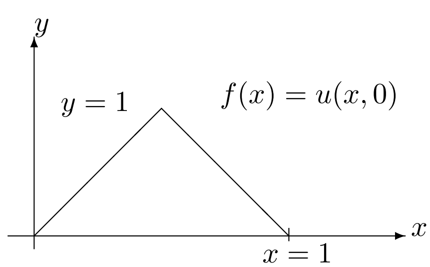
1. (10 Points) Modeling Concepts. For the following questions assume that we are considering the physical problem on a bounded domain, $x \in [0, 1]$.

   (a) Write down the heat equation and any initial and boundary conditions needed to find a unique solution.

   (b) Assume that the following graph is the initial temperature for a homogeneous heat problem with boundary conditions, $u_x(0, t) = 0$, $u_x(1, t) = 0$. Describe the physical meaning of these boundary conditions and graph the temperature profile for $t \to \infty$.



   (c) The following graph gives the only nonzero initial configuration for the heat and wave equation. Describe and/or draw the associated heat **and** wave dynamics. If there is an equilibrium state then be sure to state it.



2. (10 Points) Quick Questions

(a) The following table contains different boundary conditions for the ODE, $F'' + \lambda F = 0$, $\lambda \in [0, \infty)$. Fill in each table element with either a yes or a no.

	Boundary value problem has a cosine solution	Boundary value problem has a sine solution	Boundary value problem has a nontrivial constant solution
$F'(0) = 0,\ F(L) = 0$			
$F(0) = 0,\ F(L) = 0$			
$F'(0) = 0,\ F'(L) = 0$			
$F(0) = 0,\ F'(L) = 0$			

(b) Show that $u(x,t) = e^{it}\cos(x)$ satisfies the differential equation $iu_t = u_{xx}$.

(c) Show that $u(x,t) = \dfrac{1}{x-t}$ satisfies the differential equation $u_t + u_x = 0$.

(d) Find the time ODE consistent with the PDE $u_t = u_{xx} + F(x,t)$ such that $u(0,t) = u(L,t) = 0$.

3. (10 Points) Suppose you are given the following spatial and temporal solutions to a 1D-PDE,

$$G_n(t) = A_n e^{-t}\cos(\sqrt{\lambda_n - 1}t) + B_n e^{-t}\sin(\sqrt{\lambda_n - 1}t), \tag{1}$$

$$F_n(x) = \sin(\sqrt{\lambda_n}x), \quad \lambda_n = 2n\pi,\ n = 1, 2, 3, \ldots. \tag{2}$$

(a) Find the general solution to the PDE.

(b) Solve for any unknown constants given $u(x, 0) = 0$ and $u_t(x, 0) = g(x)$.

(c) Evaluate $\lim_{t\to\infty} u(x, t)$.

4. (10 Points) Find the three ODE consistent with $u_{tt} + .5u_t = u_{xx} + u_{yy}$.

5. (10 Points) Find the unique solution to,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{array}{c} x \in (0, \pi) \\ t \in (0, \infty) \end{array}, \tag{3}$$

$$u(0,t) = 0,\ u(\pi, t) = 0, \tag{4}$$

$$u(x,0) = 0. \tag{5}$$

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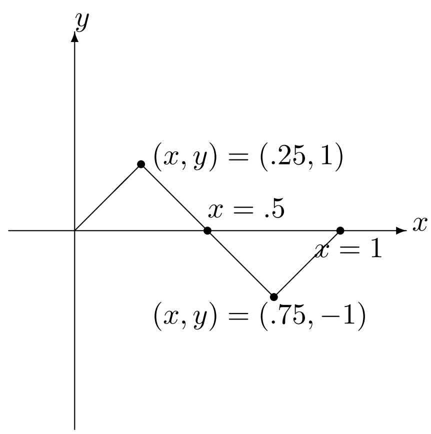
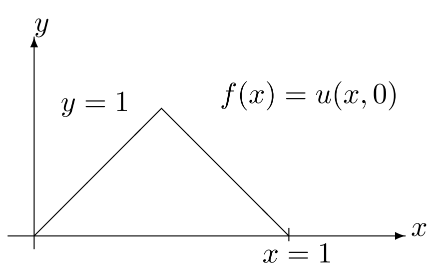
1. (10 Points) Modeling Concepts. For the following questions assume that we are considering the physical problem on a bounded domain, $x \in [0, 1]$.

   (a) Write down the wave equation <u>and</u> any initial and boundary conditions needed to find a unique solution.

   (b) Assume that the following graph is the initial displacement for a homogeneous wave problem with no initial velocity and boundary conditions, $u(0, t) = 0$, $u(1, t) = 0$. Describe the physical meaning of these boundary conditions <u>and</u> graph the temperature profile for $t \to \infty$.



   (c) The following graph gives the <u>only nonzero initial configuration</u> for the heat and wave equation. Describe and/or draw the associated heat **<u>and</u>** wave dynamics. If there is an equilibrium state then be sure to state it.



2. (10 Points) Quick Questions

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(b) Show that $u(x, t) = \ln(x^2 + y^2)$ satisfies the differential equation $u_{xx} + u_{yy} = 0$.

(c) Show that $u(x, t) = e^{it}\cos(x)$ satisfies the differential equation $iu_t = u_{xx}$.

(d) Find the time ODE consistent with the PDE $u_t = u_{xx} + F(x, t)$ such that $u_x(0, t) = u_x(L, t) = 0$.

3. (10 Points) Given the following spatial and temporal solutions to a 1D-PDE,

$$G_n(t) = A_n e^{-\lambda_n t}, \tag{6}$$

$$F_n(x) = \cos(\sqrt{\lambda_n} x), \quad \lambda_n = n, \ n = \mathbf{0}, 1, 2, 3, \ldots. \tag{7}$$

(a) Find the general solution to the PDE.

(b) Solve for any unknown constants given $u(x, 0) = f(x)$.

(c) Evaluate $\lim_{t \to \infty} u(x, t)$.

4. (10 Points) Find the three ODE consistent with $u_{tt} + .5u_t = u_{xx} + u_{yy}$.

5. (10 Points) Find the unique solution to,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{array}{c} x \in (0,\pi) \\ t \in (0,\infty) \end{array}, \tag{8}$$

$$u(0,t) = 0,\ u(\pi,t) = 0, \tag{9}$$

$$u(x,0) = f(x), \tag{10}$$

$$u_t(x,0) = 0. \tag{11}$$

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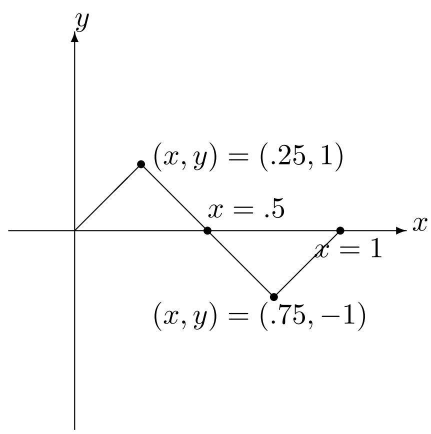
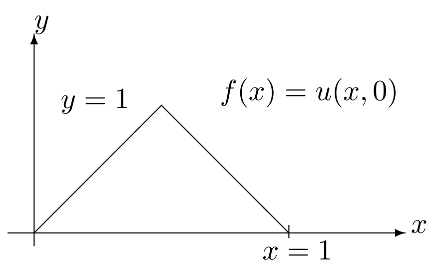


   (c) The following graph gives the only nonzero initial configuration for the heat and wave equation. Describe and/or draw the associated heat **and** wave dynamics. If there is an equilibrium state then be sure to state it.



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(c) Show that $u(x,t) = e^{it}\cos(x)$ satisfies the differential equation $iu_t = u_{xx}$.

(d) Find the time ODE consistent with the PDE $u_t = u_{xx} + F(x,t)$ such that $u(0,t) = u(L,t) = 0$.

3. (10 Points) Given the following spatial solutions to a 1D-PDE,

$$F_n(x) = \cos(\sqrt{\lambda_n} x), \quad \sqrt{\lambda_n} = n, \; n = \mathbf{0}, 1, 2, \ldots, \tag{12}$$

where the temporal ODE is given by $G_n'' + \lambda_n G_n = 0$.

(a) Find the general solution to the PDE.

(b) Solve for any unknown constants given $u(x, 0) = f(x)$.

(c) Evaluate $\lim_{t\to\infty} u(x, t)$.

4. (10 Points) Find the three ODE consistent with $u_{tt} + .5u_t = u_{xx} + u_{yy}$.

5. (10 Points) Find the unique solution to,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad \begin{array}{c} x \in (0,\pi) \\ t \in (0,\infty) \end{array}, \tag{13}$$

$$u_x(0,t) = 0,\ u_x(\pi,t) = 0, \tag{14}$$

$$u(x,0) = f(x). \tag{15}$$