MATH 332 - Linear Algebra June 17, 2009

Homework 2, Summer 2009 **Due**: June 23, 2009

Solutions to Linear Systems - Linear Independence - Spanning Sets

1. Given the following non-homogeneous linear system,

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6. \end{aligned}$$

Describe the solution set of the previous system in parametric vector form, and provide a geometric comparison with the solution to the corresponding homogeneous system.

2. Let,

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \tag{1}$$

(a) Show that the equation $\mathbf{Ax} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b} \in \mathbb{R}^3$.

(b) Describe the set of all $\{b_1, b_2, b_3\}$ for which $\mathbf{Ax} = \mathbf{b}$ *does* have a solution.

3. Define,

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 5 & 3 \\ -4 & 7 \\ 9 & -2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 22 \\ 20 \\ 15 \end{bmatrix}. \tag{2}$$

(a) Do the columns of $\mathbf{A}$ span $\mathbb{R}^4$?

(b) Is $\mathbf{b}$ a linear combination of the columns of $\mathbf{B}$?

4. Let,

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 9 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}. \tag{3}$$

(a) Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ form a linearly independent set?

(b) Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$?

5. Do the columns of,

$$\mathbf{A} = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}, \tag{4}$$

form a linearly independent set? Justify your answer.