Background

units SI vs. Gaussian (cgs)

why Gaussian?

- no ϵ_0, μ_0
- factors of 4π, c appear in logical place
- E, B fields are in same units, relativistically equivalent
- much literature in Gaussian: must know both

why not Gaussian, why SI?

- SI is more convenient for numerical, experimental evaluation.

Force law (cgs)

$$\vec{F}_{12} = \frac{q_1 q_2}{r^2}\hat{e}_r \quad \longrightarrow \quad \frac{e^2}{r^2}$$

SI

$$= \frac{1}{4\pi\epsilon_0} e^2/r^2$$

think of dimensions in terms of defining eqns.

- expressions are often in groups

$\therefore$ $e^2/r^2 \rightarrow$ force in cgs

main tool for translations $\boxed{e^2 \rightarrow e^2/4\pi\epsilon_0}$

Appx D: numerical conversion to gaussian (not recommended)

Appx E: conversion in terms of equations (better)

convert expression to SI, then evaluate numbers

Notation

Heald/Marion

grad $\rightarrow \vec{\nabla}$

div $\rightarrow \vec{\nabla}\cdot$

curl $\rightarrow \vec{\nabla}\times$

old fashioned, but emphasizes physical meaning.

Maxwell's equations (vacuum)

SI

$$\nabla\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\nabla\cdot\vec{B} = 0$$

Faraday

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Ampere corrected

$$\nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} + \mu_0\vec{J}$$

$$\vec{D} = \varepsilon_0\varepsilon_r\vec{E} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0\mu_r}\vec{B} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Gaussian

$$\nabla\cdot\vec{E} = 4\pi\rho$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

$$\vec{D} = \varepsilon\vec{E} = \vec{E} + 4\pi\vec{P}$$

$$\vec{H} = \frac{1}{\mu}\vec{B} = \vec{B} - 4\pi\vec{M}$$

Review of Maxwell eqns. (HM, Melia)

coulomb's law

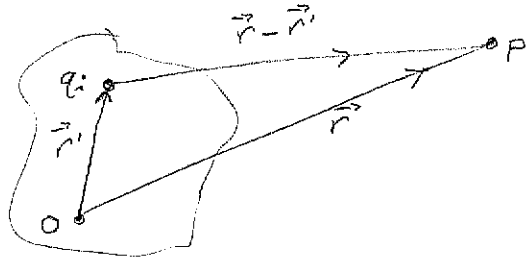
$$\vec{E} = \frac{q'}{r^2}\,\hat{e}_r$$

q' = source charge
$\vec{E}$ = force/unit charge

EM is a field theory
- the field itself has it's own reality:
  $|E|^2, |B|^2 \propto$ energy density or pressure
  linear, angular momentum.
- Max. Eq → fields created by charges, Lorentz force → charg[e] response to fields

EM field is linear: fields of diff't source pts add.

$$\vec{E}(\vec{r}) = \sum_i \frac{q(r_i')}{|\vec{r}-\vec{r}_i'|^2}\,\hat{e}_{r-r_i'}$$



electric flux

$$\Phi_E \equiv \int_{\Delta S} \vec{E}\cdot d\vec{a}$$

Gauss' law

$$\oint_S \vec{E}\cdot d\vec{a} = \underbrace{4\pi}_{\text{from } 4\pi \text{ sr}}\, q_{encl}$$

e.g. $q = e$ at origin
surface = sphere, rad = [R]

$$\oint_S \vec{E}\cdot d\vec{a} = \int_{4\pi} \frac{e}{R^2}\cdot R^2\cdot d\Omega = 4\pi e$$

differential form:

read appx A for vector calc and/or div, grad, curl book.

divergence thm

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V \vec{\nabla}\cdot\vec{E}\, dv$$

w/

$$\int_V \rho\, dv = q_{encl}$$

$$\rightarrow \vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

since Coulomb field is conservative, i.e.

$$\text{Work} = \oint e\vec{E}\cdot d\vec{l} = 0$$

by Stokes thm $\oint \vec{E}\cdot d\vec{l} = 0 \rightarrow \vec{\nabla}\times\vec{E} = 0$

true only in static case

i.e. $\nabla\times(\nabla f) = 0$ always

Since $\nabla\times\vec{E} = 0 \rightarrow$ write E in terms of a potential

$$\phi(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{l}$$

like pot. energy/chg.

close loop $\rightarrow 0$

∴ inverse

$$\vec{E} = -\vec{\nabla}\phi$$

and Poisson's eqn reads

$$\vec{\nabla}\cdot\nabla\phi = \boxed{\nabla^2\phi = -4\pi\rho}$$

Faraday's Law

EMF in a circuit induced by a changing magnetic flux

$$EMF = -\frac{1}{c}\frac{d\Phi_m}{dt}$$

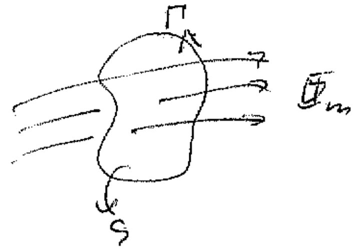
$\hookrightarrow$ from Gauss units.

this links the E and B field:

$$EMF \equiv \oint_\Gamma \vec{E}\cdot d\vec{l}$$

(normally $= 0$ b/c static field is conservative)

$$\Phi_m = \int_S \vec{B}\cdot\hat{n}\,da$$



circulation

$$\rightarrow \oint_\Gamma \vec{E}\cdot d\vec{l} = -\frac{1}{c}\int \frac{\partial\vec{B}}{\partial t}\cdot\hat{n}\,da$$

by Stokes thm:

$$\rightarrow \int_S (\nabla\times\vec{E})\cdot\hat{n}\,da$$

i.e. $\nabla\times\vec{E} = \lim_{a\to 0}\frac{1}{a}\oint_\Gamma \vec{E}\cdot d\vec{l}$

and $$\underline{\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}}$$ diff'l form of Faraday.

valid in general for EM <u>fields</u> not just circuits

Conservation of charge + continuity eqn.

current flowing through a surface is

$$I = \int_S \vec{J} \cdot d\vec{a} = \int_S \vec{J} \cdot \hat{n} \, da$$

q

for a closed surface

$$I_{out} = \oint_S \vec{J} \cdot \hat{n} \, da = -\frac{dq}{dt}$$

b/c chg is leaving

$$= -\frac{d}{dt} \oint_V \rho \, dv$$

now use div. thm

$$\int_V \nabla \cdot \vec{J} \, dv = -\int_V \frac{\partial \rho}{\partial t} \, dv$$ for S fixed in space.

$$\rightarrow \vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$ continuity eqn.

charge is absolutely conserved,
in plasmas often keep track of ion ρ_i and elect ρ_e separately.

$$\rightarrow \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = W$$ = ionization rate, source term.

Magnetic field:

no magnetic monopoles proven, so

$$\oint_S \vec{B} \cdot d\vec{a} = 0 \quad \rightarrow \quad \vec{\nabla} \cdot \vec{B} = 0$$

Lorentz Force

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

note v/c factor highlights rel. strengths of interaction of E, B on a charge.

For uniform veloc → currents

Biot-Savart

$$\vec{B} = \frac{1}{c} \frac{Q \vec{v} \times \hat{r}}{r^2}$$

single chg Q moving w/ vel $\vec{v}$

for a current,

$$\vec{B} = \frac{1}{c} \oint \frac{I \, d\vec{l} \times \hat{r}}{r^2}$$

path along wire

Amperes law

$$\oint_\Gamma \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_{link}$$

Γ arb. closed path

I_{link} = sum All currents linking that loop.

$$\downarrow \qquad\qquad \downarrow$$

$$\int_S \nabla \times \vec{B} \cdot d\vec{a} = \frac{4\pi}{c} \int_S \vec{J} \cdot d\vec{a}$$

J is total current density.

$$\nabla \times \vec{B} = \frac{4\pi}{c} J$$

Maxwell's mod of Ampere's Law

steady state: $\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J}_{free}$

div: $\underbrace{\nabla \cdot (\nabla \times \vec{H})}_{0} = \frac{4\pi}{c} \nabla \cdot \vec{J}_{free}$

$\nabla \cdot \vec{J} = 0$ in steady state, by continuity, $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$!

since $\nabla \cdot \vec{D} = 4\pi\rho \rightarrow \frac{\partial \rho}{\partial t} = \frac{1}{4\pi} \nabla \cdot \frac{\partial \vec{D}}{\partial t}$

$\rightarrow \quad \nabla \cdot (\nabla \times \vec{H}) = \frac{4\pi}{c}\left(\nabla \cdot \vec{J}_{free} + \frac{\partial \rho}{\partial t}\right)$

$= \frac{4\pi}{c} \nabla \cdot \vec{J}_{free} + \frac{1}{c} \nabla \cdot \frac{\partial \vec{D}}{\partial t}$

take out div:

$$\underline{\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}}$$

↳ displac't current.

difficult to observe in circuits (need high ω)

but essent'l for EM waves.