Lecture 10 Shadowitz finish chapter 3 and begin chapter 4.

What have we covered?

1.) Forces on currents $d\vec{F} = I\, d\vec{r} \times \vec{B}$

$$= \vec{K} \times \vec{B}\, da$$

$$= \vec{J} \times \vec{B}\, d\text{volume}$$

2.) Finding $\vec{B}$ given current density

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{r}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, da' \qquad \vec{K}(x',y',z')$$

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3}\, dx'dy'dz'$$

3.) $\vec{\nabla} \cdot \vec{B} = 0$ $\qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\downarrow$$

$$\int \vec{\nabla} \times \vec{B} \cdot d\vec{a} = \int \mu_0 \vec{J} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r}$$

Helmholtz theorem says we need $\vec{\nabla} \cdot$ & $\vec{\nabla} \times$ vector function to uniquely determine it.

We left off here in Monday's lecture:

What is $\vec{\nabla} \cdot B$?

$$\vec{\nabla} \cdot \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{(\vec{r} - \vec{r}\,')^3} dx' dy' dz'$$

$$= \frac{\mu_0}{4\pi} \int \vec{\nabla} \cdot \left[\frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{(\vec{r} - \vec{r}\,')^3} \right] dx' dy' dz'$$

$$\vec{\nabla} \cdot \left(\frac{\vec{J} \times \hat{r}}{r^2} \right) = \frac{\hat{r}}{r^2} \cdot (\vec{\nabla} \times \vec{J}) - \vec{J} \cdot \left(\vec{\nabla} \times \frac{\hat{r}}{r^2} \right)$$

How do you prove this vector identity (informational)?

$$\vec{\nabla} \cdot (\vec{J} \times \vec{G}) = \vec{\nabla} \cdot \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ J_x & J_y & J_z \\ G_x & G_y & G_z \end{vmatrix} = \vec{\nabla} \cdot \left[\hat{x} (J_y G_z - G_y J_z) - \hat{y} (\right.$$

$$\vec{G} = \frac{\hat{r}}{r^2}$$

$$= \frac{\partial}{\partial x} (J_y G_z - G_y J_z) - \frac{\partial}{\partial y} (J_x G_z - G_x J_z) + \frac{\partial}{\partial z} (J_x G_y - G_x J_y)$$

So $\vec{\nabla} \cdot \vec{B} = 0$

How can the divergence of B be zero if there is a source of B (incongruous)?

Doesn't the divergence tell us about sources of the vector field (incongruous)?

The zero diverence just tells us that the vector field is not diverging say from a point or faucet.

The magnetic field does not come out of a point like the electric field. It is generated by currents which generate a "circular" field. The non-zero curl tells us the source of B.

what about $\vec{\nabla} \times \vec{B}$ (causal/creative)?

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0}{4\pi} \int \vec{\nabla} \times \left[\frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{(\vec{r} - \vec{r}\,')^3} \right] dx' dy' dz'$$

$$= \mu_0 \vec{J}(x, y, z) \text{ not } \vec{J}(x', y', z')$$

Why is J a function of the unprimed variables(incongruous)?

The infinity when r = rprime causes the integral to generate $\vec{J}(x,y,z)$ not $\vec{J}(x',y',z')$

$$\boxed{\vec{\nabla} \times \vec{B}(x,y,z) = \mu_0 \vec{J}(x,y,z)}$$

Apply stokes theorem

$$\int \vec{\nabla} \times \vec{B} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r}$$

stokes theorem

$$\boxed{\int \mu_0 \vec{J} \cdot da = \oint \vec{B} \cdot d\vec{r}}$$

Ampere's Law

What is a simple example of current density (congruos)?

Conservation of charge $\qquad \vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$

$\frac{C}{m^3}\ \frac{m}{s} \rightarrow \frac{C}{m^2 s}$ or $\frac{\text{Amps}}{m^2}$

↑ ↑ units

$\vec{J} \equiv \rho\,\vec{v}$

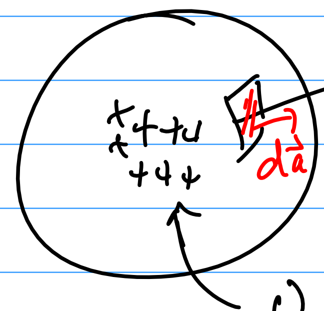
multiply by dvolume and integrate

$$\int \vec{\nabla}\cdot\vec{J}\,d\text{volume} = -\frac{\partial}{\partial t}\int \rho\,d\text{volume}$$

divergence th.

$$\oint \vec{J}\cdot d\vec{a} = -\frac{d}{dt}Q_{\text{enclosed}}$$

Stuff a bunch of plus charge in a spherical shell and let go. The charge moving out of the surface is due to the loss of charge within the surface.



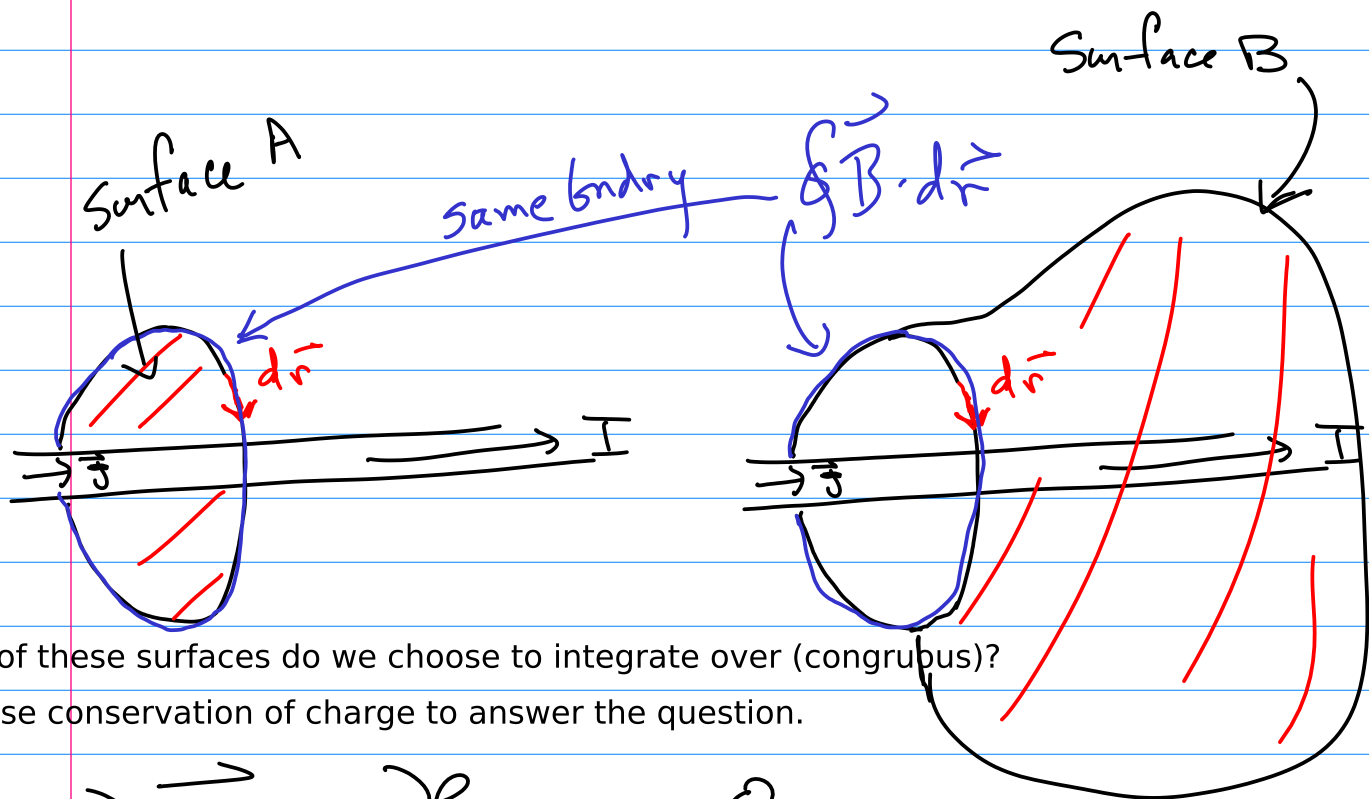
$$\oint \vec{J}\cdot d\vec{a} = -\frac{dQ_{\text{encl}}}{dt}$$

Qenclosed

Questions about $\int \mu_0 \vec{J}\cdot d\vec{a} = \oint \vec{B}\cdot d\vec{r}$?

There are infinitely many surfaces that share the same boundary line. Which one are we supposed to use (congruous)?

Example ?



Which of these surfaces do we choose to integrate over (congruous)?

Use conservation of charge to answer the question.

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\int \vec{\nabla}\cdot\vec{J}\, dvol = -\frac{\partial}{\partial t}\overbrace{\int \rho\, dvol}^{Q_{encl.}\text{ by that surface}}$$

$$\parallel$$

$$\oint \vec{J}\cdot d\vec{a} = -\frac{d}{dt}Q_{encl.} = 0 \quad \text{magnetostatics}$$

$$\parallel$$

$$\oint \vec{J}\cdot d\vec{a} = \int_{\text{surface A}} \vec{J}\cdot d\vec{a} + \int_{\text{surface B}} \vec{J}\cdot d\vec{a} = 0$$

The flux thru each surface is the same in magnitude so it doesn't matter which surface we choose to apply Amp's law with.

What are we going to cover?

1) Ampere's law examples

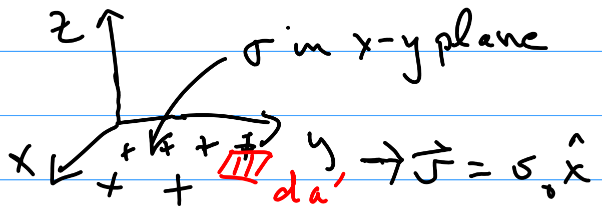
2.) More Biot-Savart examples

3.) Back to electrostatics to find $\vec{\nabla} \times \vec{E} = ?$ $\quad \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

4.) Define voltage and potential energy in electrostatics.

1.) Infinite planar sheet of charge moving at constant speed v and charge density sigma.

How do you calculate B (congruous)?



Method 1

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}\, da' \; : \quad \vec{K}(x',y',z') = \sigma v_0 \hat{x}$$

$$da' = dx'\, dy'$$

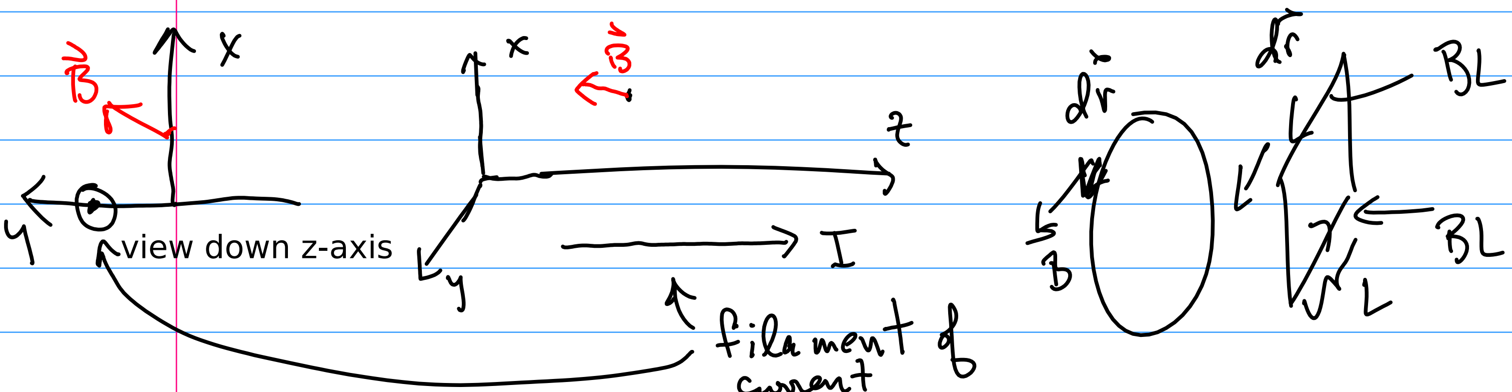
$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}\,' = x'\hat{x} + y'\hat{y}$$

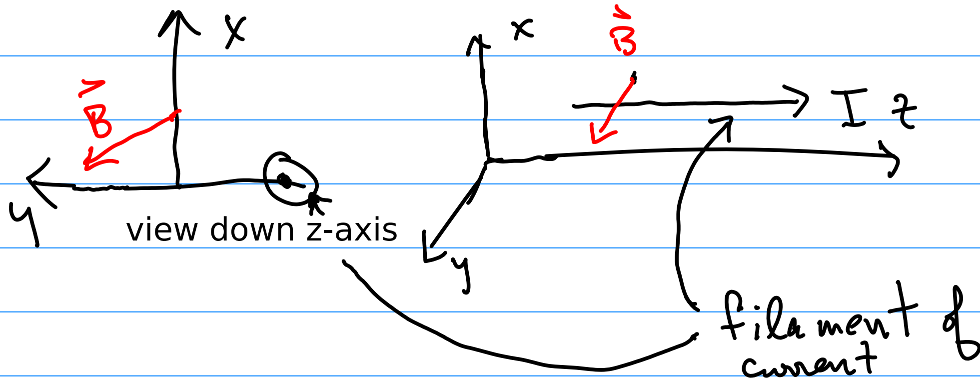
$$\vec{K} \times \vec{\mathfrak{r}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \sigma v_0 & 0 & 0 \\ \mathfrak{r}_x & \mathfrak{r}_y & \mathfrak{r}_z \end{vmatrix}$$

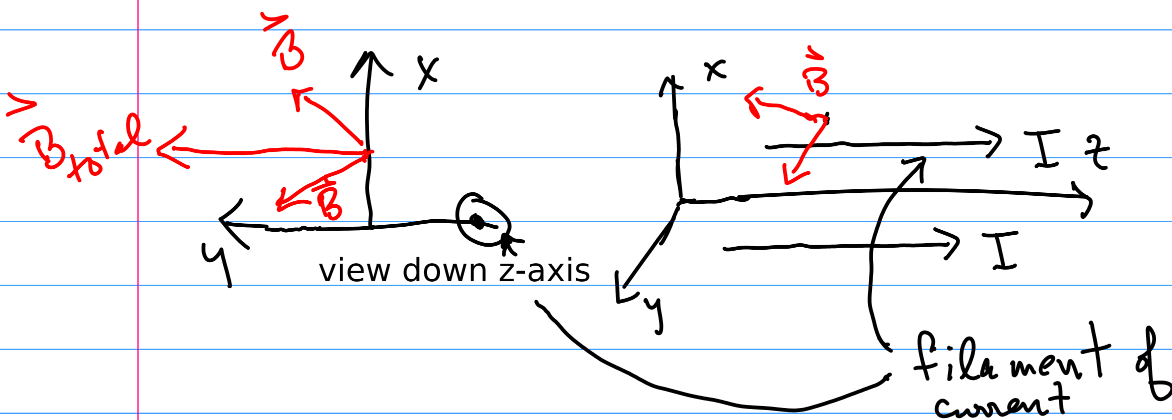
Method 2 . . . . . . use Ampere's law $\quad \mu_0 \int \vec{J} \cdot d\vec{a} = \oint \vec{B} \cdot d\vec{r} \sim 2BL$

What do we know and what do we want to find out (informational)?



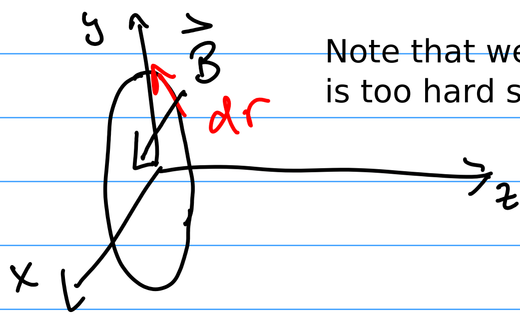
choose a symmetrically locate filament of current in the y-z plane



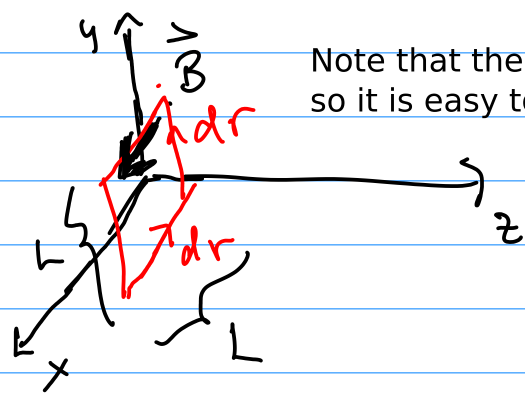


By adding current filaments symmetrically located we get the surface current. By adding their B's we see B must point parallel to the y-z plane and in opposite directions above and below the sheet of current in the y-z plane.

Now we can easily apply Ampere's law. First choose a path, a circle in the x-y



Note that we need the angle between B and dr. This is too hard so lets try a square path



Note that the angle between B and dr is either 0 or 90 degrees so it is easy to calculate $\vec{B}\cdot d\vec{r} = |\vec{B}|\,|d\vec{r}|\,\underbrace{\cos 0}_{1}$ or $\underbrace{\cos 90}_{0}$

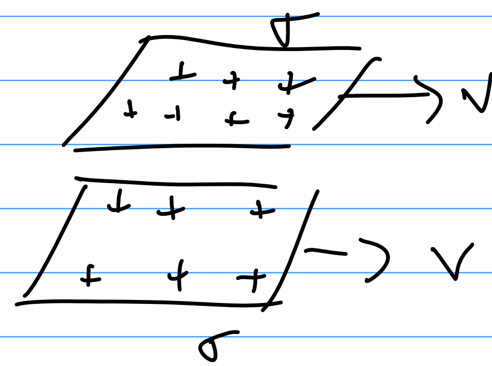
$$\oint \vec{B}\cdot d\vec{r} = BL + \phi + BL + \phi = \mu_0 \int \vec{J}\cdot d\vec{a}$$

sheet of current

$$\mu_0 \left(\text{current enclosed by square}\right) = \mu_0 \int_0^L K\,dx = \mu_0 K L$$

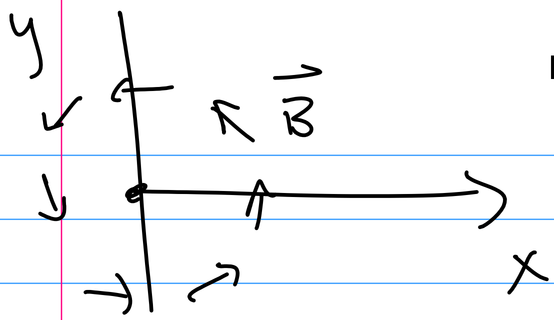
$$2BL = \mu_0 K L \quad \text{or} \quad \boxed{B = \frac{\mu_0 K}{2}}$$

2.) Two infinite planar sheets of charge moving.



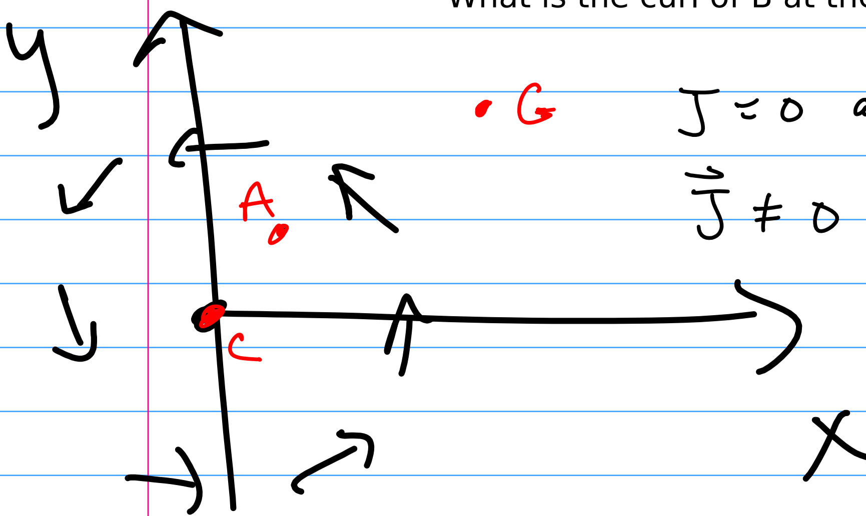
What do we know and what do we want to find out (informational)?

Use the superposition principle to apply what we learned about 1 sheet of current to solve this problem.



Infinite wire on the z axis carrying current I

What is the curl of B at these points? $\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$



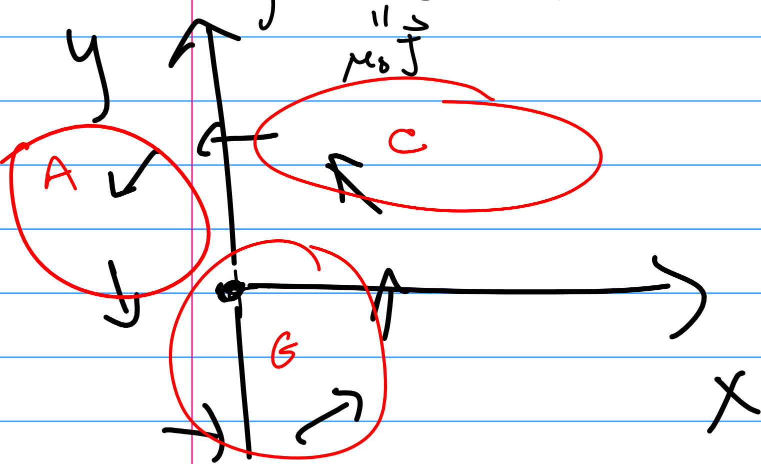
$J=0$ at points A & G so $\vec{\nabla}\times\vec{B}=0$

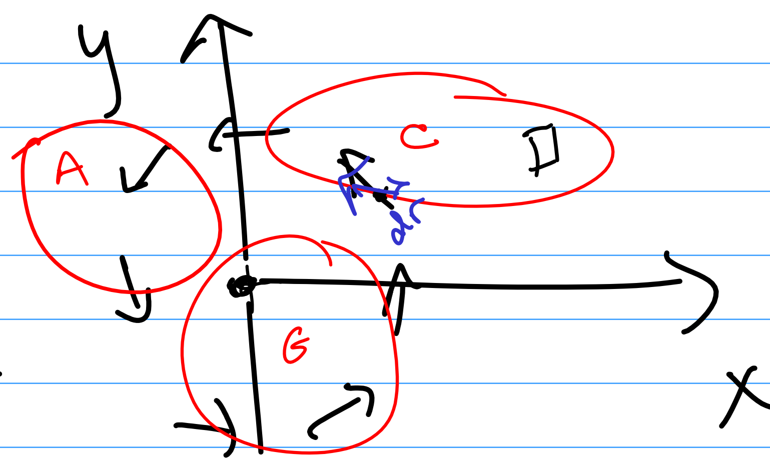
$\vec{J}\neq 0$ at point C so $\vec{\nabla}\times\vec{B}\neq 0$

Why did you say that (informational)?

stokes

What is the $\int \underbrace{\vec{\nabla}\times\vec{B}}_{=\mu_0\vec{J}}\cdot d\vec{a}$ in these surfaces $=\oint \vec{B}\cdot d\vec{r}$





$$\int \vec{\nabla}\times\vec{B}\cdot d\vec{a}=\oint\vec{B}\cdot d\vec{r}$$

$$\int \mu_0\vec{J}\cdot d\vec{a}=\oint\vec{B}\,d\vec{r}$$

Since J is zero within surfaces A and C the LHS is 0.

Since J is non zero within surface G the LHS $\neq \phi$